Special Lectures V
Theoretical Astronomy & Astrophysics

1. Cosmic & Galactic Evolution and Origin of Matter

Particle Cosmology
Dynamical Large-Scale Structure (LSS) Formation
Evolution of Matter (Chemical Evolution)

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How was the beginning of the Universe?

**Unified theory (Superstring Theory, M Theory, Quantum Gravity …)
needs higher dimension of space-time
and predicts “quantum fluctuations” of the Baby Universe.**

Creation and Annihilation
of the Baby Universe

Symmetry was dynamically broken, and the space-time underwent cosmic phase transition to grow exponentially (INFLATION).

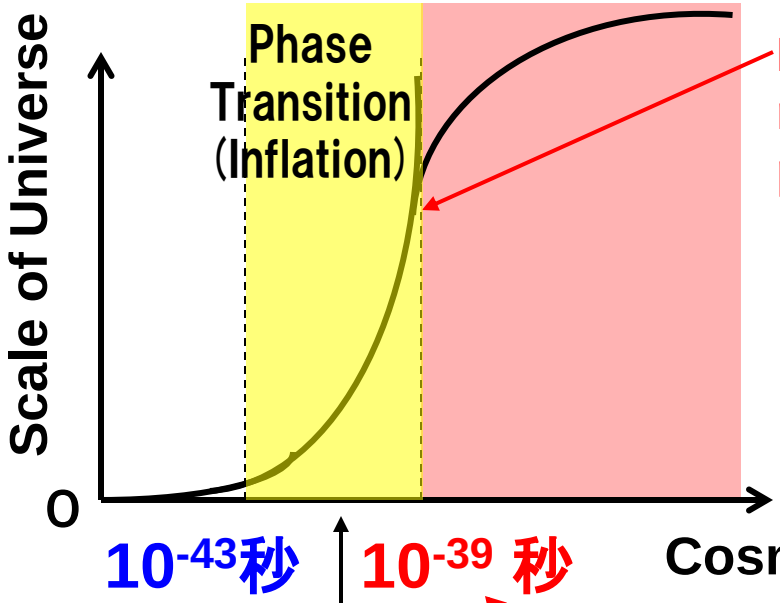
➡ **Universe!**

The Universe is one of multiverse.

Quantum Uncertainty Principle

$$\Delta x \, \Delta p > \hbar/2$$

Universe could not be created without $\hbar$!



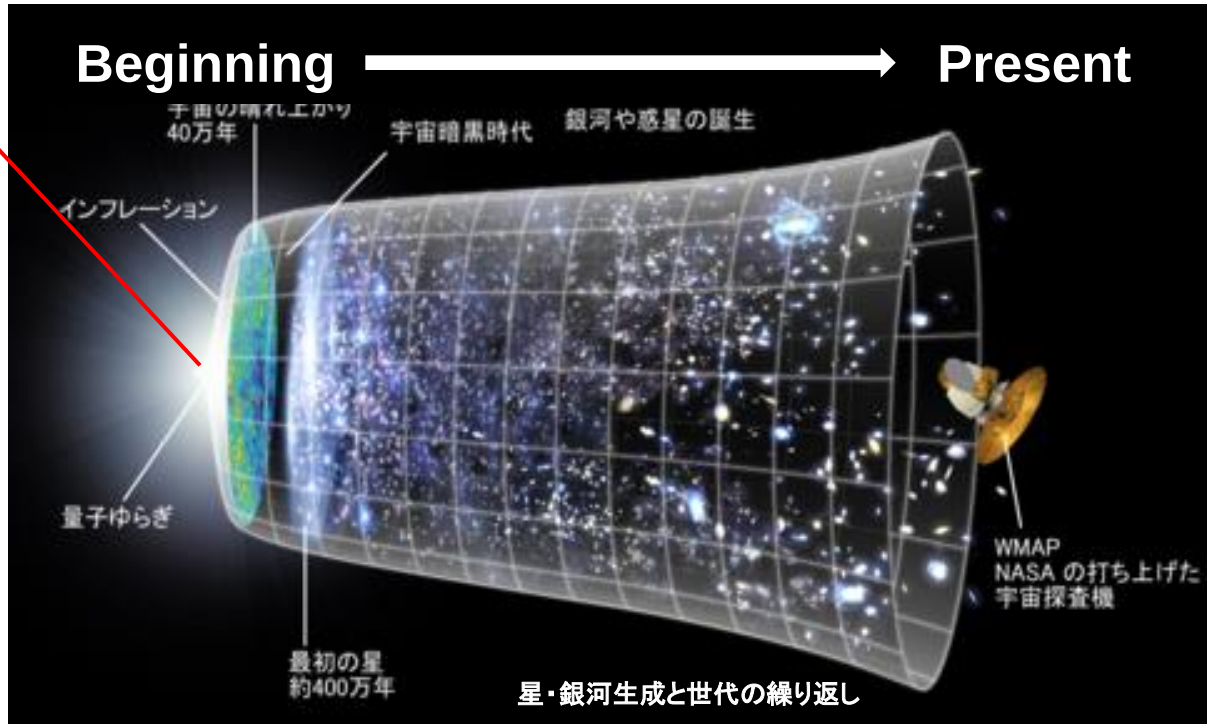
Latent heat of the phase transition was released to create elementary particles, i.e. photon (γ), neutrino (ν), electron (e), quarks (q) …

➡ Hot Big-Bang Fireball

Vacuum is not empty (P. M. Dirac) after 10^{-39}秒 !

Symmetry was broken dynamically.

Before 10^{-43}, space-time was born out of the quantum fluctuations.



March 17, 2014

arXiv:submit/0934323 [astro-ph.CO]

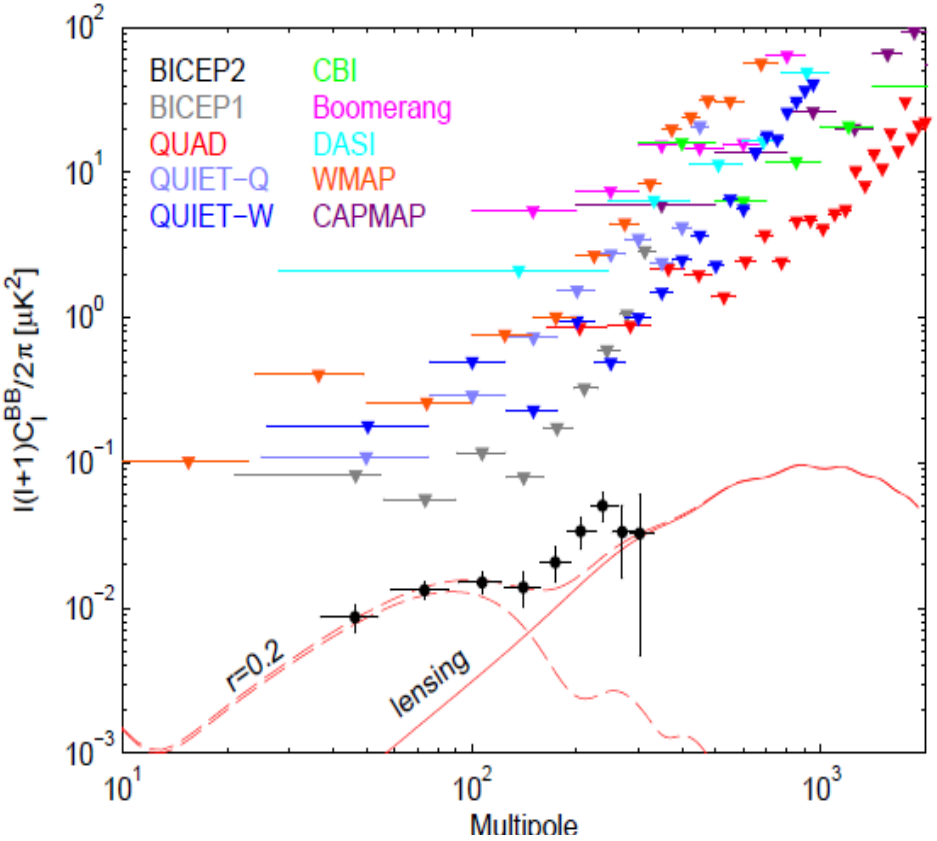
BICEP2 team for the measurement of CMB polarization anisotropies announced possible discovery of cosmic background GW mode!

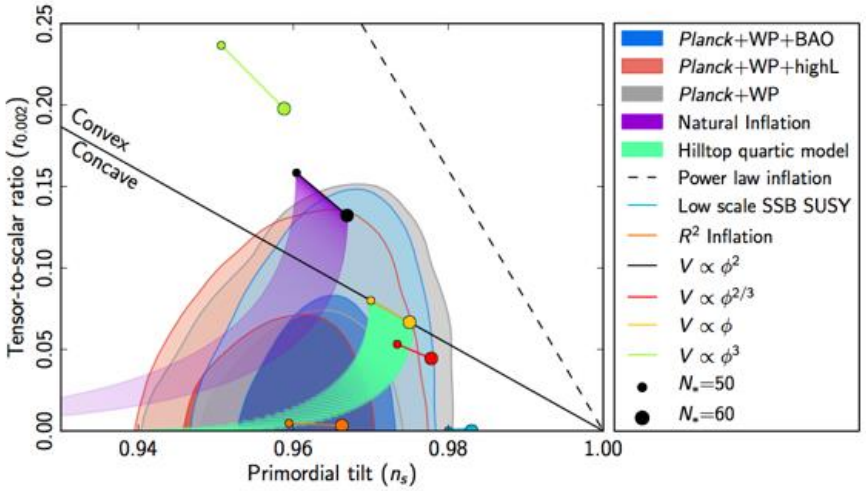
Tensor/Scalar
= 0.2 (+0.07, -0.05)

Caotic Inflation
(A. Linde, 1983)

- Background GW (A. Starobinsky 1979)
- Old inflation
  (K. Sato, A. Guth 1981)

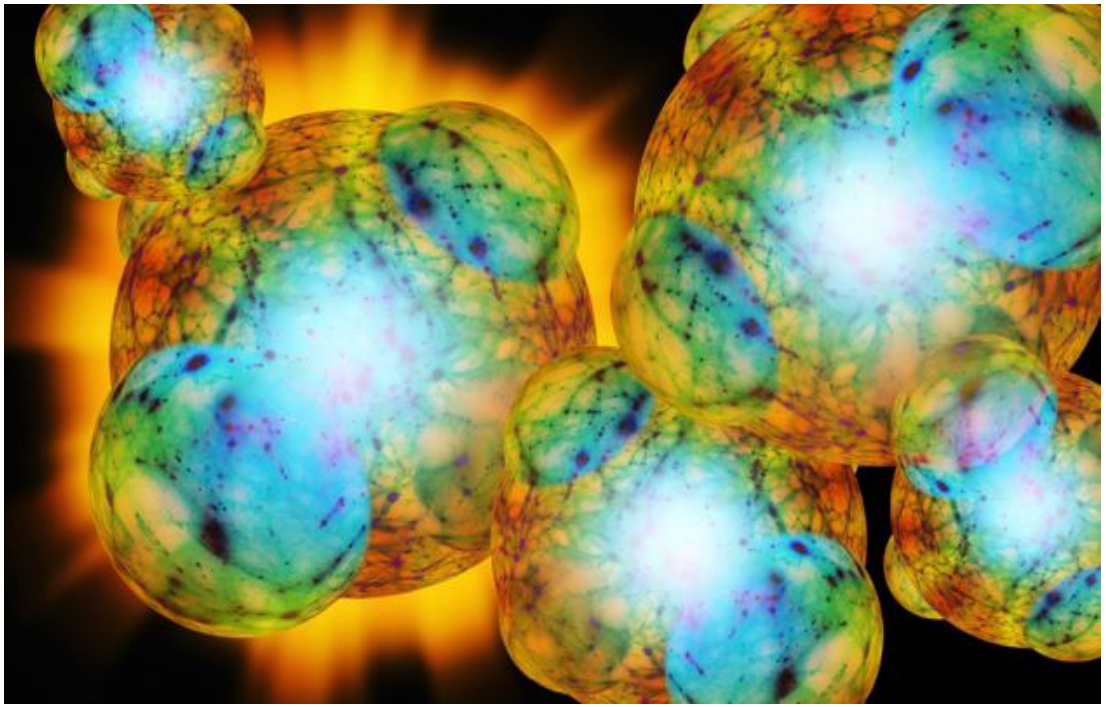
T/S is larger than prediction of superstring theory !?





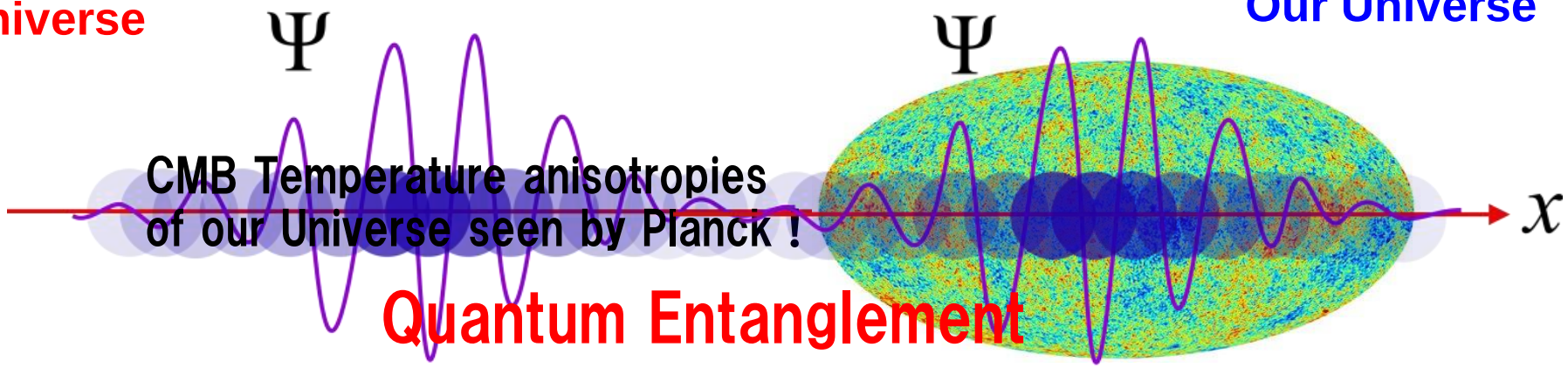
http://www.preposterousuniverse.com/blog/
2014/03/16/gravitational-waves-in-the-cosmic-microwave-background/

Birth of the Universe in Caotic Inflation ⇒ Multiverse



Wave Fnct. of Neighboring Universe

Wave Fnct. of Our Universe



Quantum Entanglement

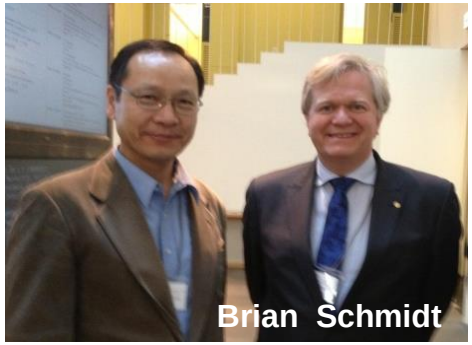
Dark Flow to be discovered in Type Ia SNe ?

An EVIDENCE for Quantum Entanglement ?

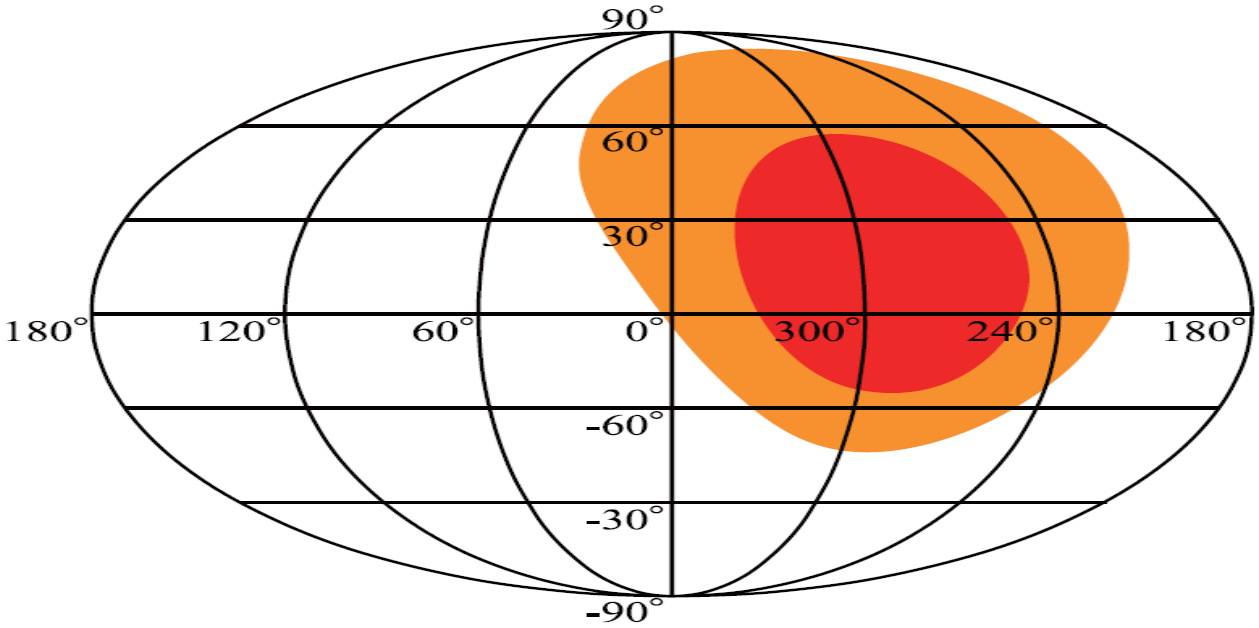
G.J. Mathews, B. Rose, P. Garnavich (Notre Dame), D. Yamazaki, T. Kajino (Tokyo),
Astrophys. J. 827 (2016), 60.

T. K. Grant J. Mathews

Peter Garnavich



T. K.

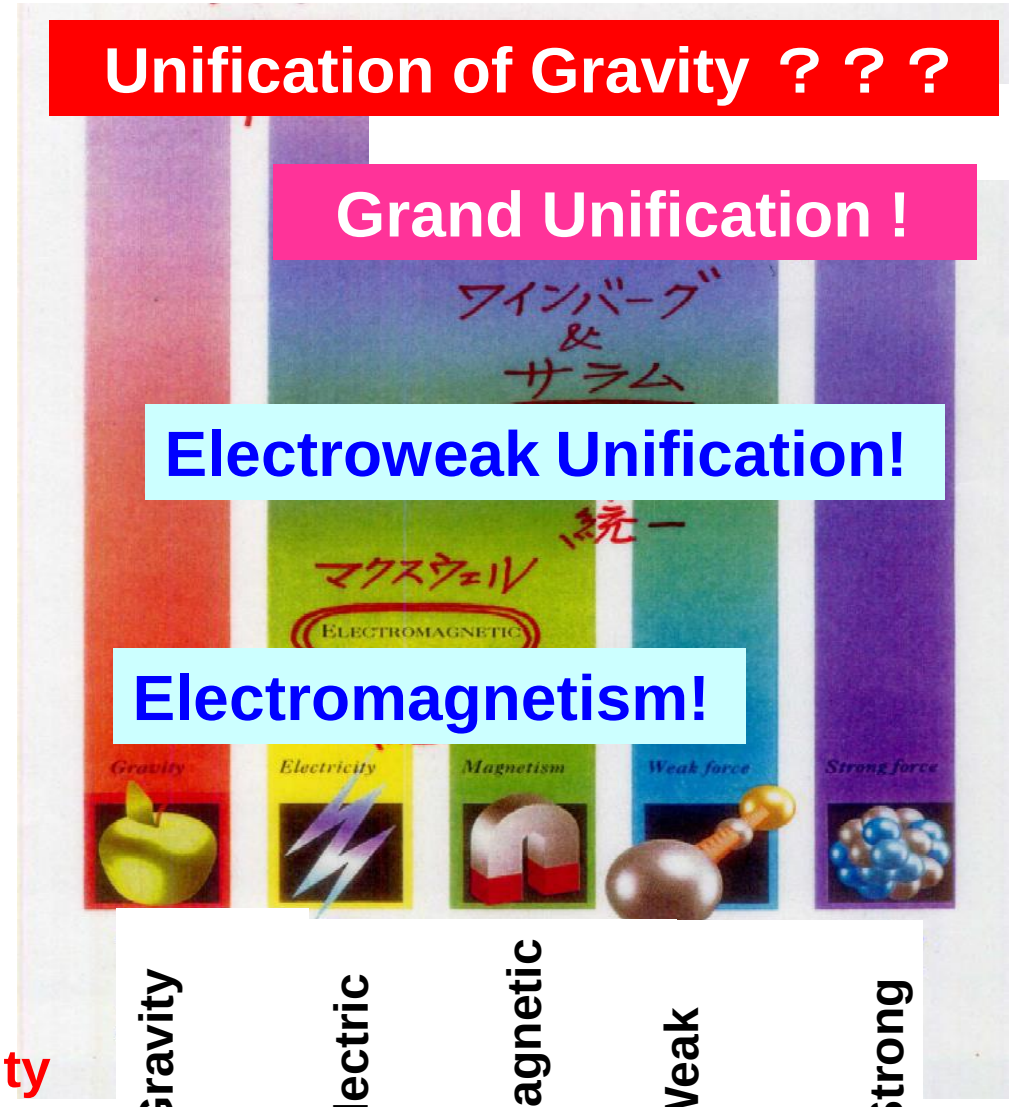


Ultimate Challenge of Modern Science !

:- is to construct Unified Theory of Fundamental Forces, and to resolve the mystery of the beginning and evolution of the Universe!

- **Electromagnetism**
  Maxwell (1864)
- **Electroweak Unification**
  Weinberg and Salam (1973)
- **Grand Unification !**
  Gauge Theory, unfinished !
- **Unification of Gravity ???**
  Superstring, SUSY, Supergravity

Need EXTRA DIMENSION ?



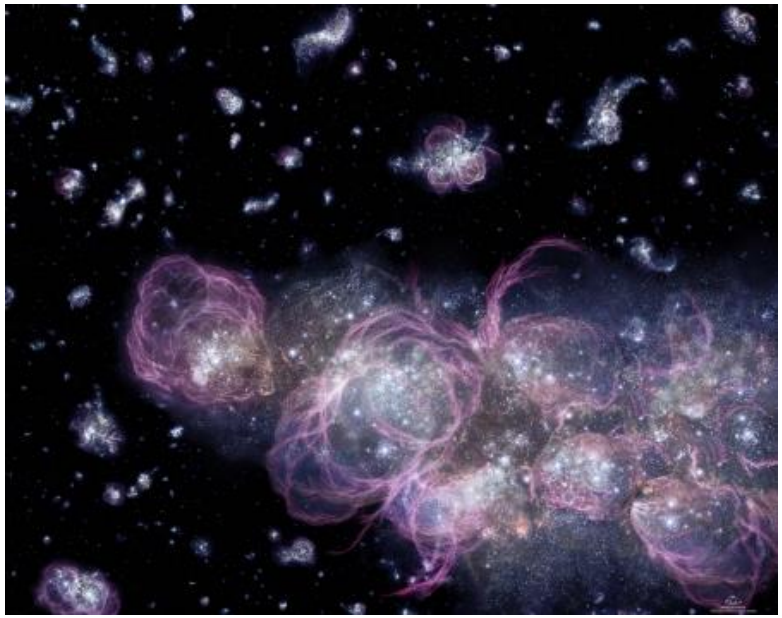
Superstring Theory – 10 Dimension

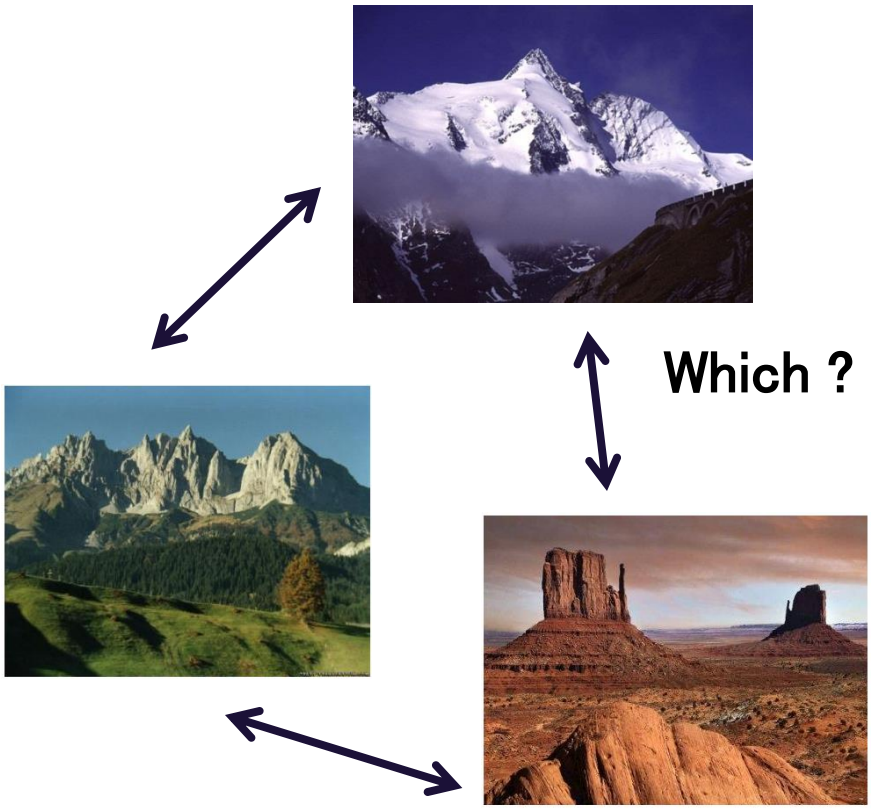
A Unified Theory, Landscape Theory

- There are ~ 10^{500} vacua in string theory.
- Vacuum energy ρ_v may be positive or even negative.
- Some of them have $\rho_v << M_P$.

Universe ➡ **Multiverse**

Quest for observational signal of Multiverse!

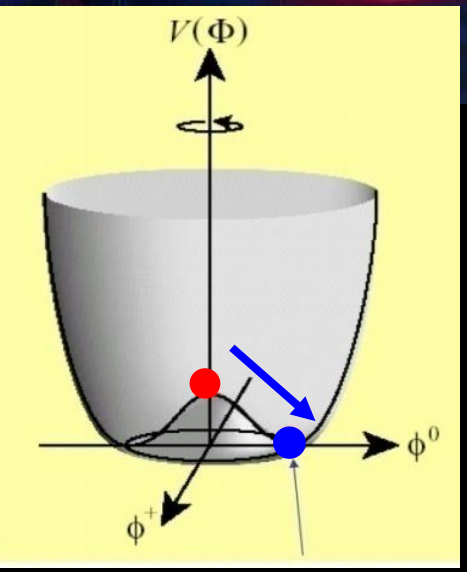




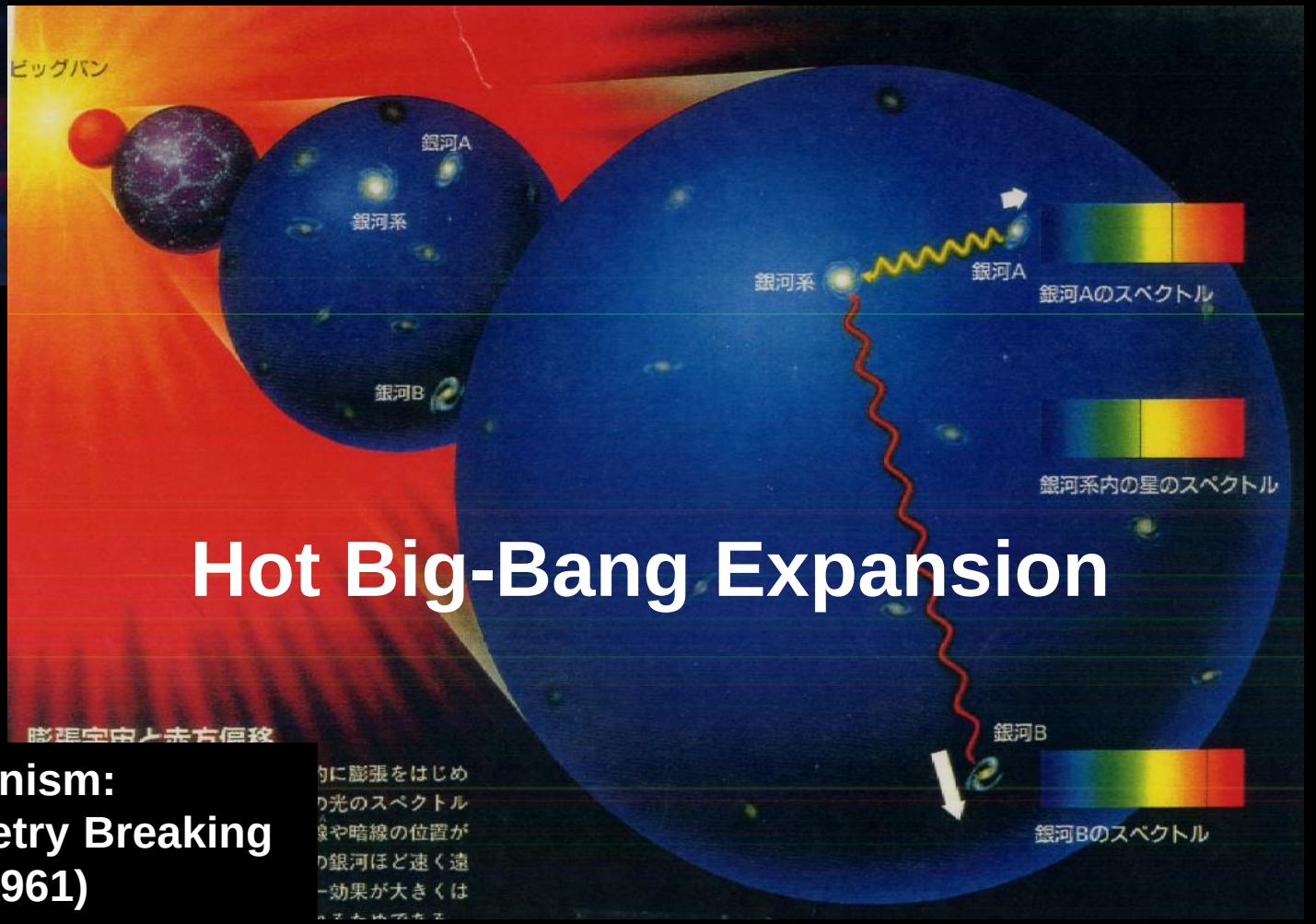
Why could our Universe grow?

Symmetry was BROKEN !

Baby Universe



Higgs mechanism:
Spontaneous Symmetry Breaking
Y. Nambu (1961)

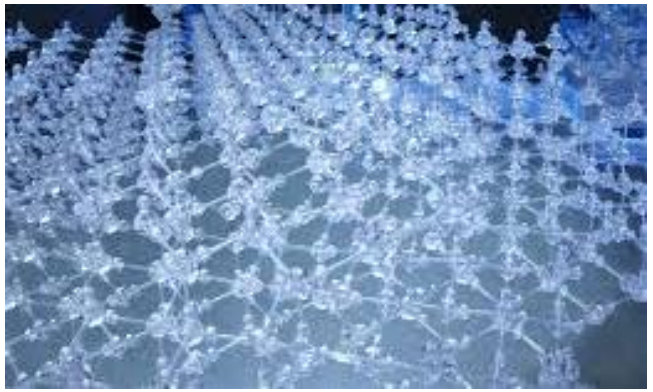


Hot Big-Bang Expansion

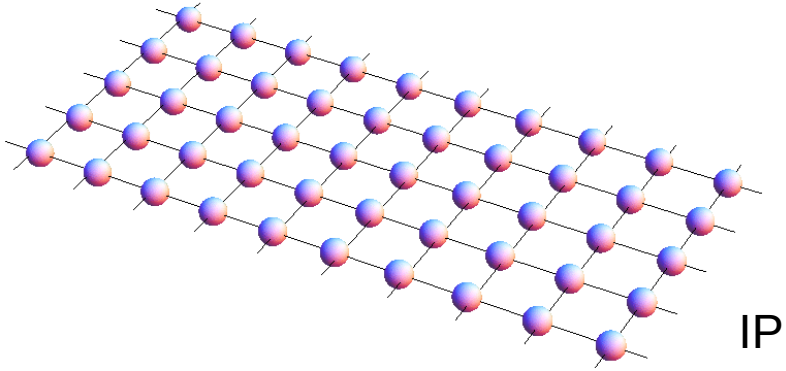
Spontaneous Symmetry Breaking !

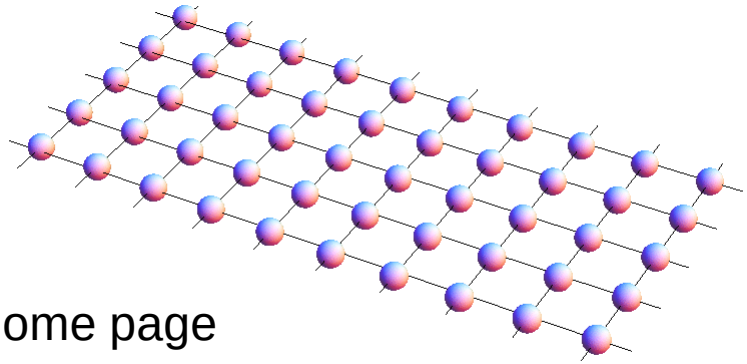
Liquid water has a continuous symmetry for spatial translation.
However, ice (solid water) has a periodic structure. ➡ **Dynamical symmetry is spontaneously broken!**

Phase Transition



Y. Nambu discovered in 1961 that if a continuous symmetry is spontaneously broken, wave is created. Wave is equivalent to a quantum particle, called Nambu-Goldstome boson. Nambu predicted that the number of N-G bosons is the same as the degrees of freedom of breaking symmetries.



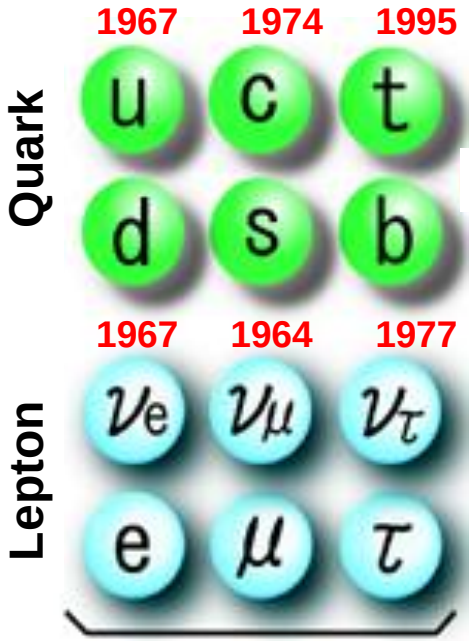


IPMU home page

Standard Theory of Elementary Particles and Fields

Matter particles to form the Universe, and Field Particles to bring forces.

Matter Particles



3 generation

M. Kobayashi
H. Masukawa (1973)

Gauge Particles

Unification ?

?

U(1) x SU(2) x SU(3)

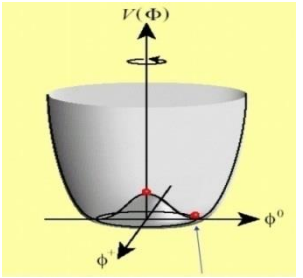
Higgs Particles

To be found ?

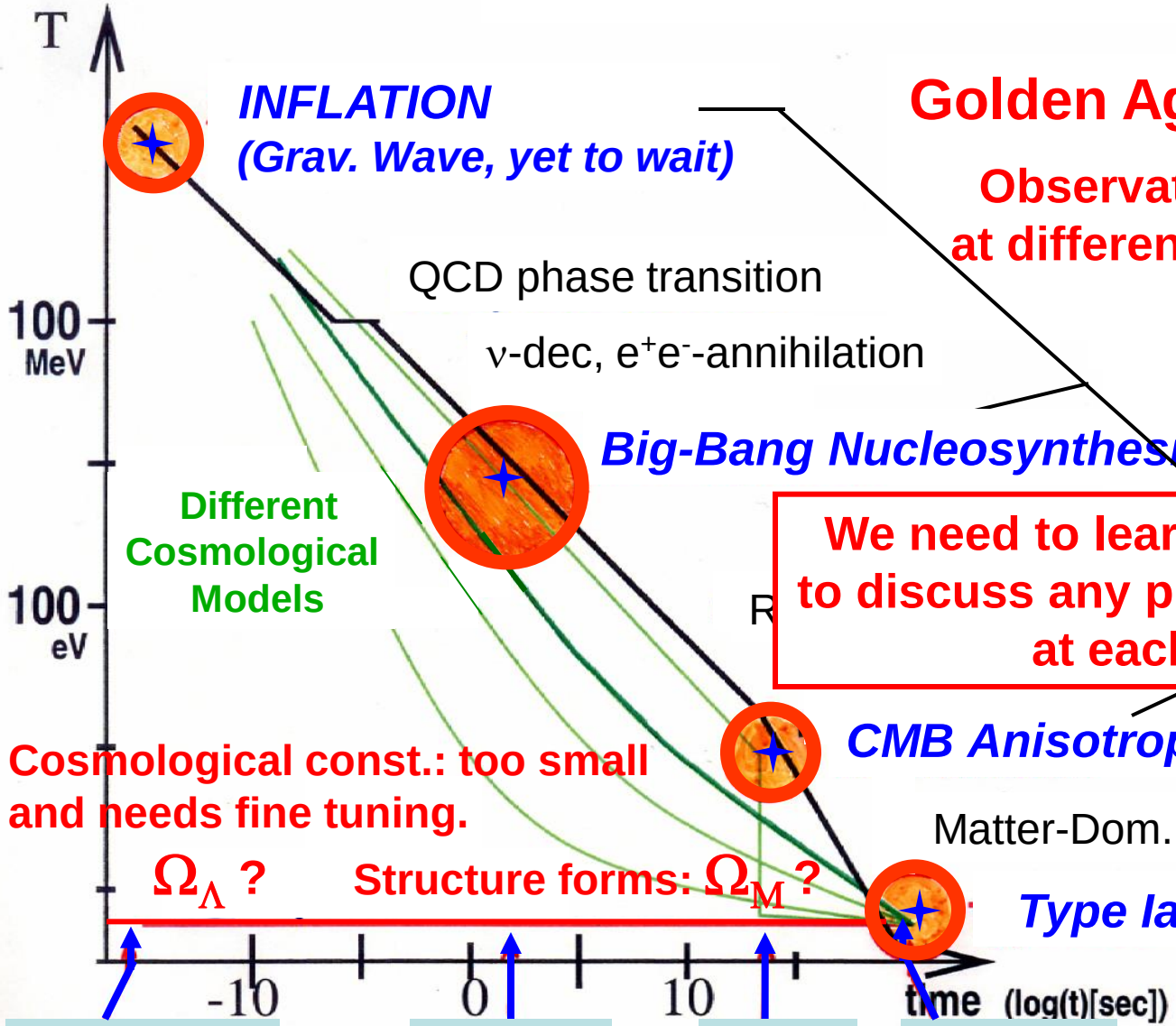
F. Englert (1964) P. Higgs

Y. Nambu
(1961)

Spontaneous Symmetry Breaking



Thermal History of the Universe



Golden Age of Cosmology

Observational Signatures at different cosmic epochs !

We need to learn the cosmology to discuss any physical processes at each epoch!

Cosmological const.: too small and needs fine tuning.

Ω_Λ ?

Structure forms: Ω_M ?

10^{-43} sec

3 min

10^5 y

1-10 Gy

OUTLINE

A Challenge of the Century
Why is the Universe flat and accelerating?
$\Omega_B + \Omega_{CDM} + \Omega_\Lambda = 1$?

- **What is the CDM, $\Omega_{CDM} = 0.268$?**
  Elementary particles, to be discoverd? Astron.Observation?
- **What is DARK ENERGY, $\Omega_\Lambda = 0.683$?**
  Unified theory of extra-dimension?
- **Baryonic Matter, $\Omega_B = 0.049$, perfectly known ?**
  Big-Bang, Stellar and Supernova Nucleosynthesis?

This Lecture

- **CMB Anisotropies ($t \sim 3.8 \times 10^5$ y) constrain cosmic evolution from RD – Last Photon Scatt. – MD – Λ-dominated Universe.**
- **Redshift-magnitude relation of the Type Ia SNe ($t \sim$ 1-10 Gy) constrains turn over from Cosmic Deceleration – Acceleration.**
- **Big-Bang and Supernova Nucleosynthesis is a CANDLE to look at dark side of the Universe.**

Basics of Theoretical Astronomy and Astrophysics – 2
Oct. 3, 2016

Basics of the Standard Cosmology

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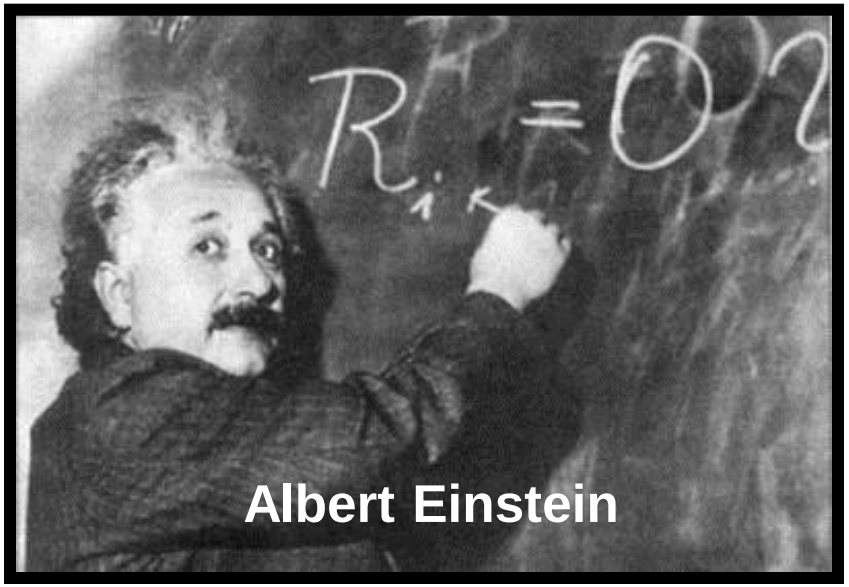
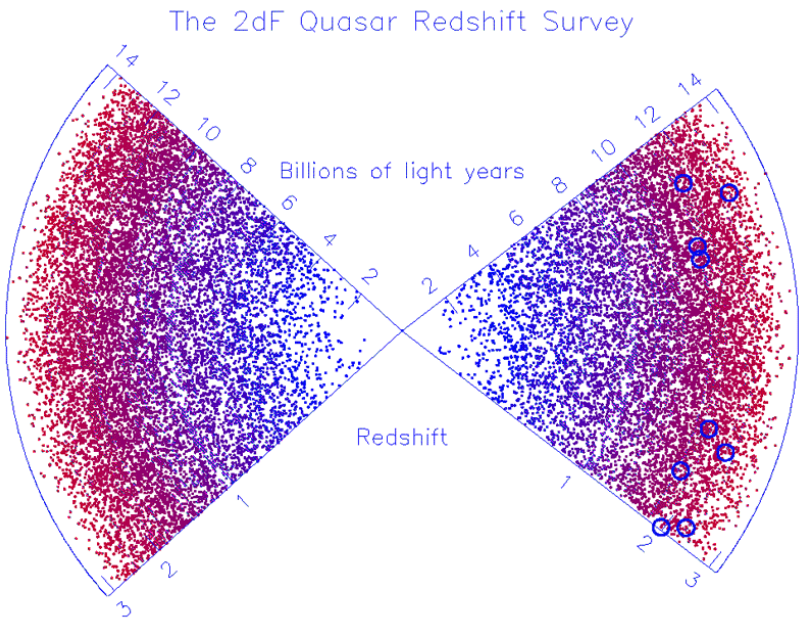
SCIENTIFIC GOAL is
to elucidate the tight coupling between the frontline of cosmology and astrophysics.

Einstein Cosmology 1915	Standard Big-Bang Model 1948



Standard Big-Bang Cosmology

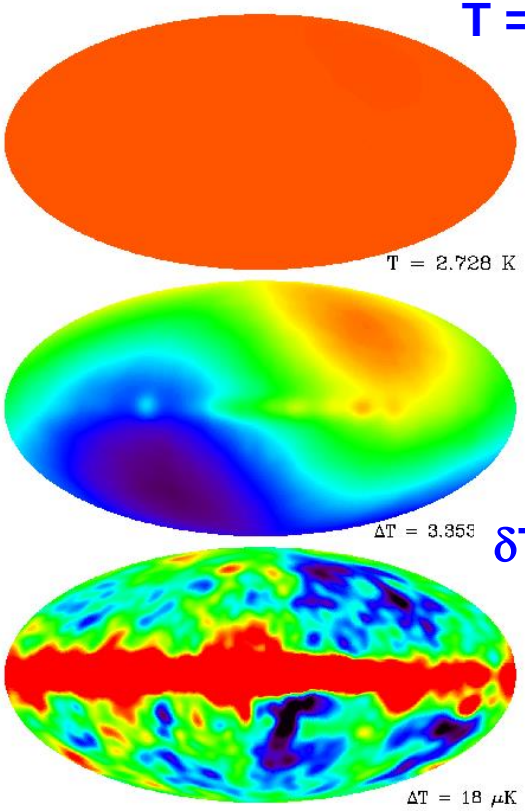
The Universe is homogeneous and isotropic in a large enough scale.



2dF Quasar (Matter) Distribution:

Homogeneous

T = 2.728 K



$\delta T/T < 10^{-5}$

Sky Maps of CMB:

Isotropic

Robertson-Walker Metric: HOMOGENEOUS & ISOTROPIC

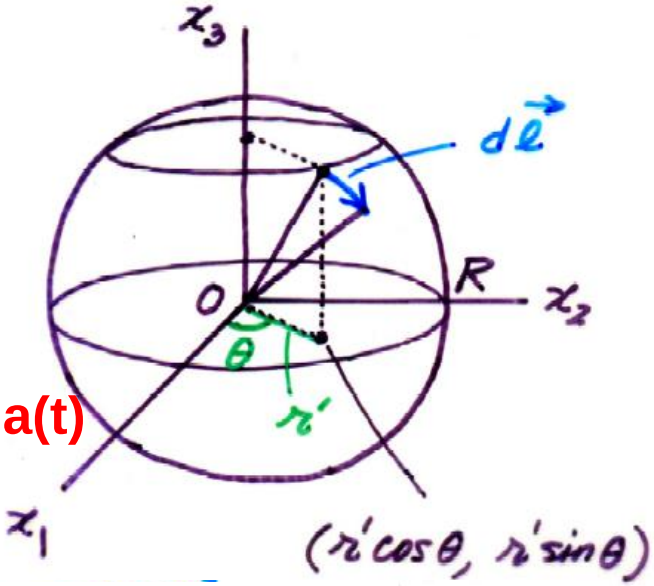
Isotropy

$$ds^2 \equiv -g_{\mu\nu}dx^\mu dx^\nu \overset{c\equiv 1}{=} dt^2 - 2\overset{0}{\overset{\|}{g_{0i}}}dx^0 dx^i - g_{ij}dx^i dx^j = dt^2 - d\vec{\ell}^{\,2}$$

Positively curved S2, embedded in 3D Euclidean space

Radius "R" → Scale Factor "a(t)"

Homogeneity R(t) = a(t)



$$R^2 = x_1^2 + x_2^2 + x_3^2$$

$$0 = x_1dx_1 + x_2dx_2 + x_3dx_3$$

$$d\vec{\ell}^{\,2} = dx_1^2 + dx_2^2 + dx_3^2$$

$$= dx_1^2 + dx_2^2 + \frac{(x_1dx_1 + x_2dx_2)^2}{R^2 - (x_1^2 + x_2^2)}$$

$$= r'^2 d\theta^2 + \frac{R^2 dr'^2}{R^2 - r'^2}$$

$$\equiv R^2\left(r^2 d\theta^2 + \frac{dr^2}{1-r^2}\right)$$

$$0 \leqq r \equiv \frac{r'}{R} \leqq 1$$

Positively and Negatively curved S2

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2\right\}$$

$k = \pm 1,\ 0\ (\text{Flat})$

Real World is S3

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right\}$$

General Relativity

R(t) → a(t) = scale factor

(1) In the limit of Weak Gravity, Einstein equation turns out to be Newtonian Gravity,

(2) Covariance,

(3) Constant velocity of light.

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = 8\pi\, G T^{\mu\nu} + \Lambda g^{\mu\nu}$$

$$R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\mu\lambda} + \Gamma^\lambda_{\eta\lambda}\Gamma^\eta_{\mu\nu} - \Gamma^\lambda_{\eta\nu}\Gamma^\eta_{\mu\lambda}$$

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\beta}\left\{\partial_\nu g_{\beta\mu} + \partial_\mu g_{\beta\nu} - \partial_\beta g_{\mu\nu}\right\}$$

$$g_{\mu\nu} = \begin{bmatrix} -1 & & & \\ & \dfrac{a^2(t)}{1-kr^2} & & \\ & & a^2(t) r^2 & \\ & & & a^2(t) r^2 \sin^2\theta \end{bmatrix}$$

$$T^\mu{}_\nu = \begin{bmatrix} -\rho & & & \\ & p & & \\ & & p & \\ & & & p \end{bmatrix}$$

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = 8\pi\, G T^{\mu\nu} + \Lambda g^{\mu\nu}$$

time-time component:

Dynamical Eq. of Motion

$$\dot{a}^2 + k = 8\pi G/3\, \rho a^2 + \Lambda/3\, a^2 \qquad (1)$$

space-space component:

$$2a\ddot{a} + \dot{a}^2 + k = -8\pi G p a^2 + \Lambda a^2 \qquad (2)$$

$\frac{\mathbf{d}}{\mathbf{dt}}$[ax(1)] & (2):

3rd Law of Thermodynamic (Energy Conservation)

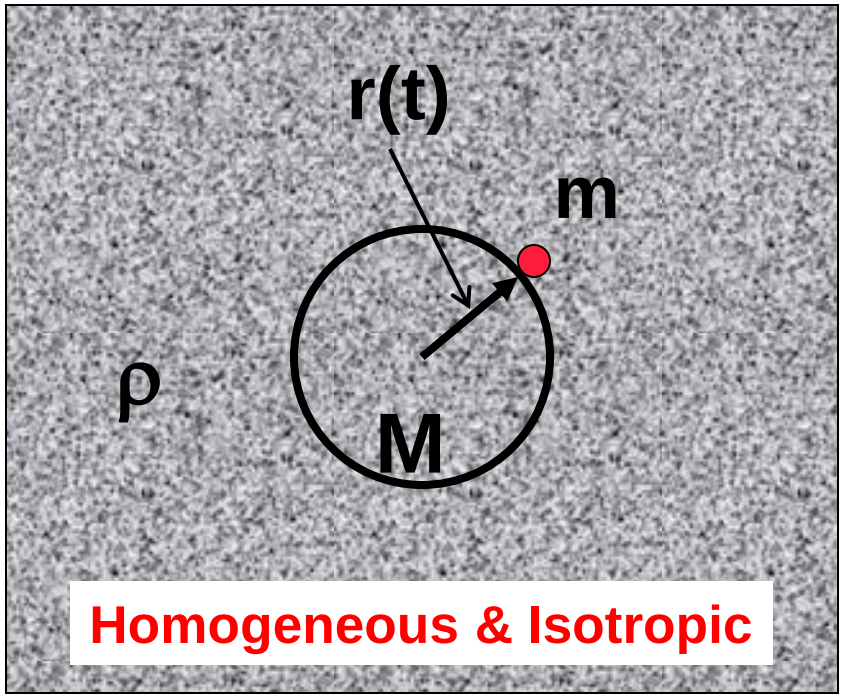
$$\frac{d}{dt}(\rho a^3) + p\frac{d}{dt}a^3 = 0 \qquad (3)$$

EOS (Equation of State)

$$p = \rho/3 \text{ (rel)} \qquad (4)$$

Newtonian Equation

Birkoff's Theorem:
Gravity due to mass interior to an arbitrary sphere.



$$E = \frac{1}{2}mv^2 - \frac{GmM}{r}$$

$$\frac{1}{2}mv^2 = \frac{Gm[(4/3)\pi\rho r^3]}{r} + E$$

$\downarrow$ x $1/2mr^2$

$$\left(\frac{v}{r}\right)^2 = \frac{8}{3}\pi G\rho + \frac{2E}{mr^2}$$

$M = 4/3\pi\rho r^3$

Einstein Equation

$$G^{00} = 8\pi\, GT^{00} + \Lambda g^{00}$$

Friedmann Eq.

$$H^2 = \frac{8}{3}\pi G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

Cosmological Consant.

$$H^2 = H_0^2\left(\frac{\Omega_\gamma}{a^4} + \frac{\Omega_M}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda\right)$$

$T \propto a^{-1}$ $\quad \Omega \longrightarrow 1$ at $t = t_0$, $a_0 = 1$

Newtonian Equation

Hubble parameter

H = v/r —k = E/m

$$\left(\frac{v}{r}\right)^2 = \frac{8}{3}\pi G\rho + \frac{2E}{mr^2}$$

$\Omega_\alpha = \rho_\alpha/\rho_C$ $\quad \rho_C = 3H_0^2/8\pi G$

a = r = scale factor

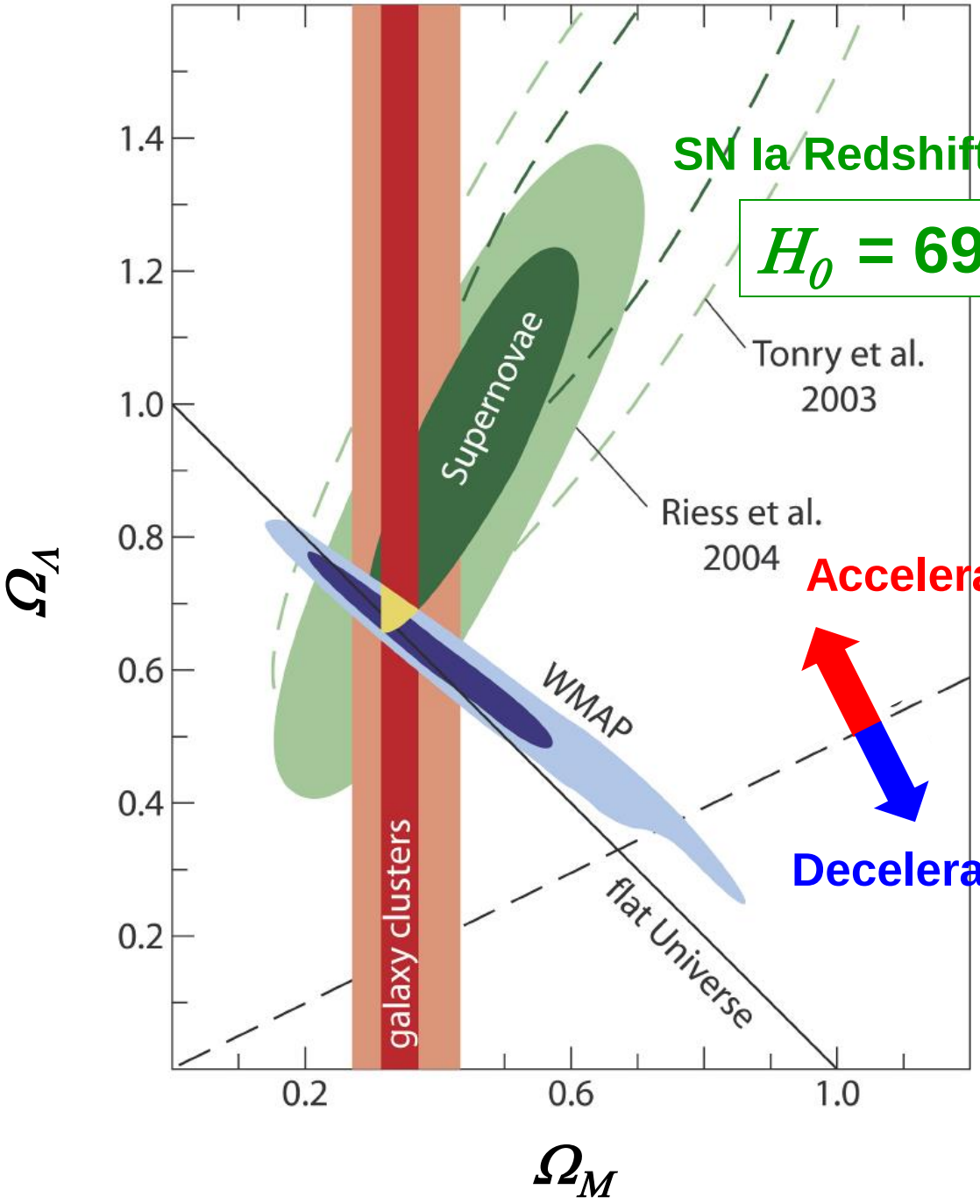
Deceleration parameter

$$q_0 = -(d^2r/dt^2)/rH^2 = [\Omega_{CDM}/2 - \Omega_\Lambda]$$

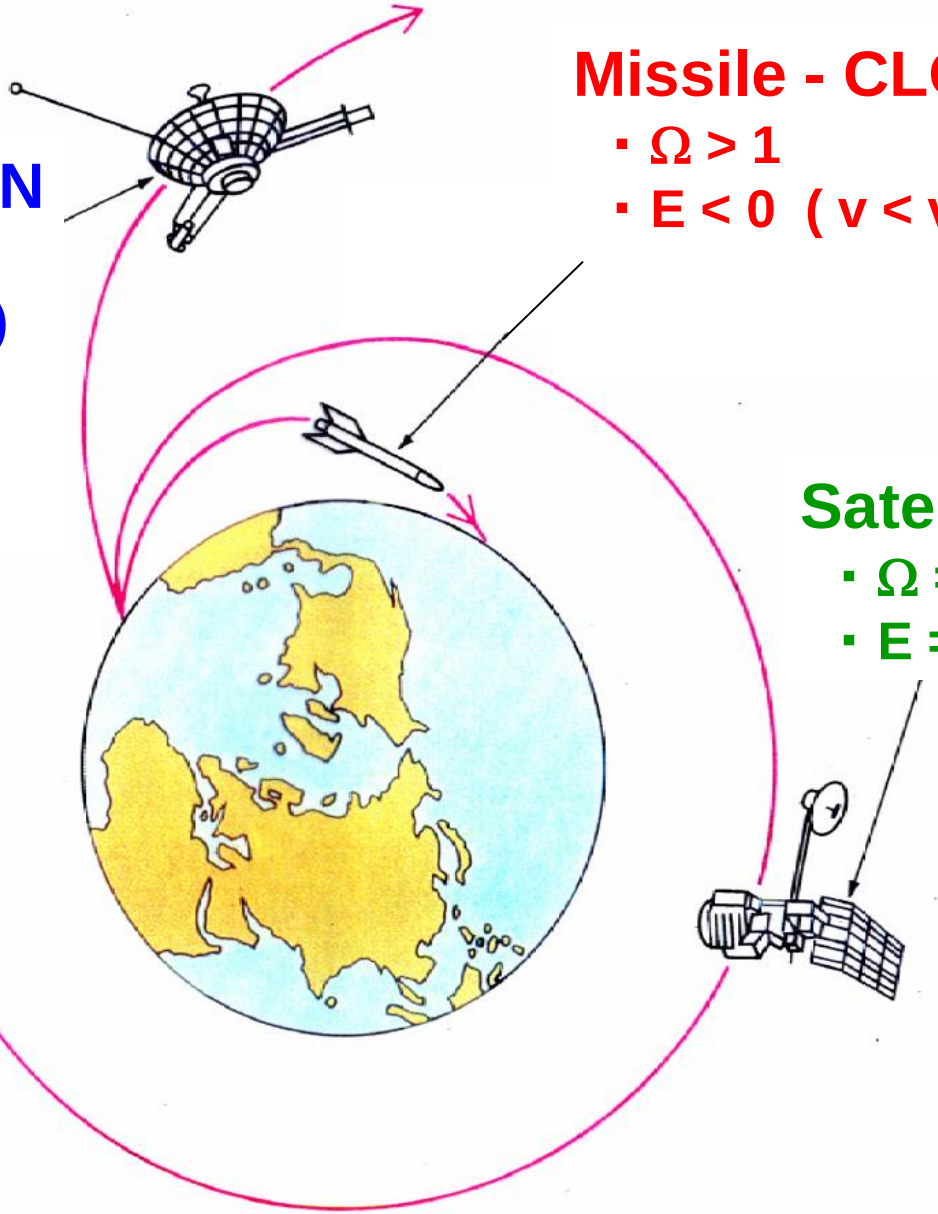
0.3 0.7

$\Omega_{CDM}/2 < \Omega_\Lambda$

acceleration !

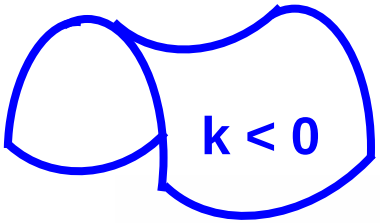

SN Ia Redshift-Magnnitude Rel.
H_0 = 69 km/s/Mpc
Tonry et al. 2003
Riess et al. 2004
Supernovae
WMAP
galaxy clusters
flat Universe
Accelerating
Decelerating
$\Omega_M/2 - \Omega_\Lambda = 0$
Ω_Λ
Ω_M
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.2
0.6
1.0

Newtonian Orbits: OPEN or CLOSED ?



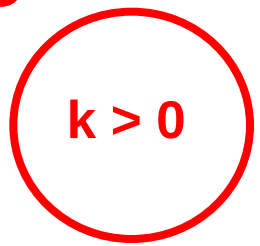
Explorer - OPEN

- $\Omega < 1$
- $E > 0$ ($v > v_{esc}$)



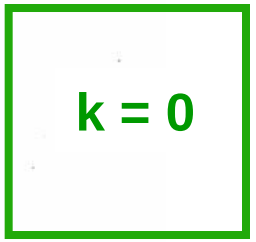
Missile - CLOSED

- $\Omega > 1$
- $E < 0$ ($v < v_{esc}$)



Satellite -MARGINAL

- $\Omega = 1$
- $E = 0$ ($v = v_{esc}$)



Escaping Velocity

$v_{esc} = 11.2$ km/s

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8}{3}\pi G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$\frac{d}{dt}(\rho a^3) + p\frac{d}{dt}a^3 = 0$$

EOS

$\Lambda = 0$:

Matter Dominated Era

1) k = 0 $\left(\frac{\dot{a}}{a}\right)^2 \propto a^{-3}$ $(\rho \propto ma^{-3})$ $a^{1/2}da \propto dt$ $a \propto t^{2/3}$

2) k =-1 < 0 $\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G/3\ ma^{-3} + a^{-2} \rightarrow a^{-2}$ (for large a)

$da \propto dt$ $a \propto t$

3) k =+1 > 0 $\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G/3\ ma^{-3} - a^{-2} \rightarrow$ 0 (at some $a = a_s$) & bounce later.

$\Lambda > 0$ and dominates:

$\left(\frac{\dot{a}}{a}\right)^2 = \Lambda/3$ $a^{-1}da \propto dt$ $a \propto \exp[(\Lambda/3)^{1/2}t]$

2016, UT Lectures
Basics of Theoretical Astronomy and Astrophysics

Report

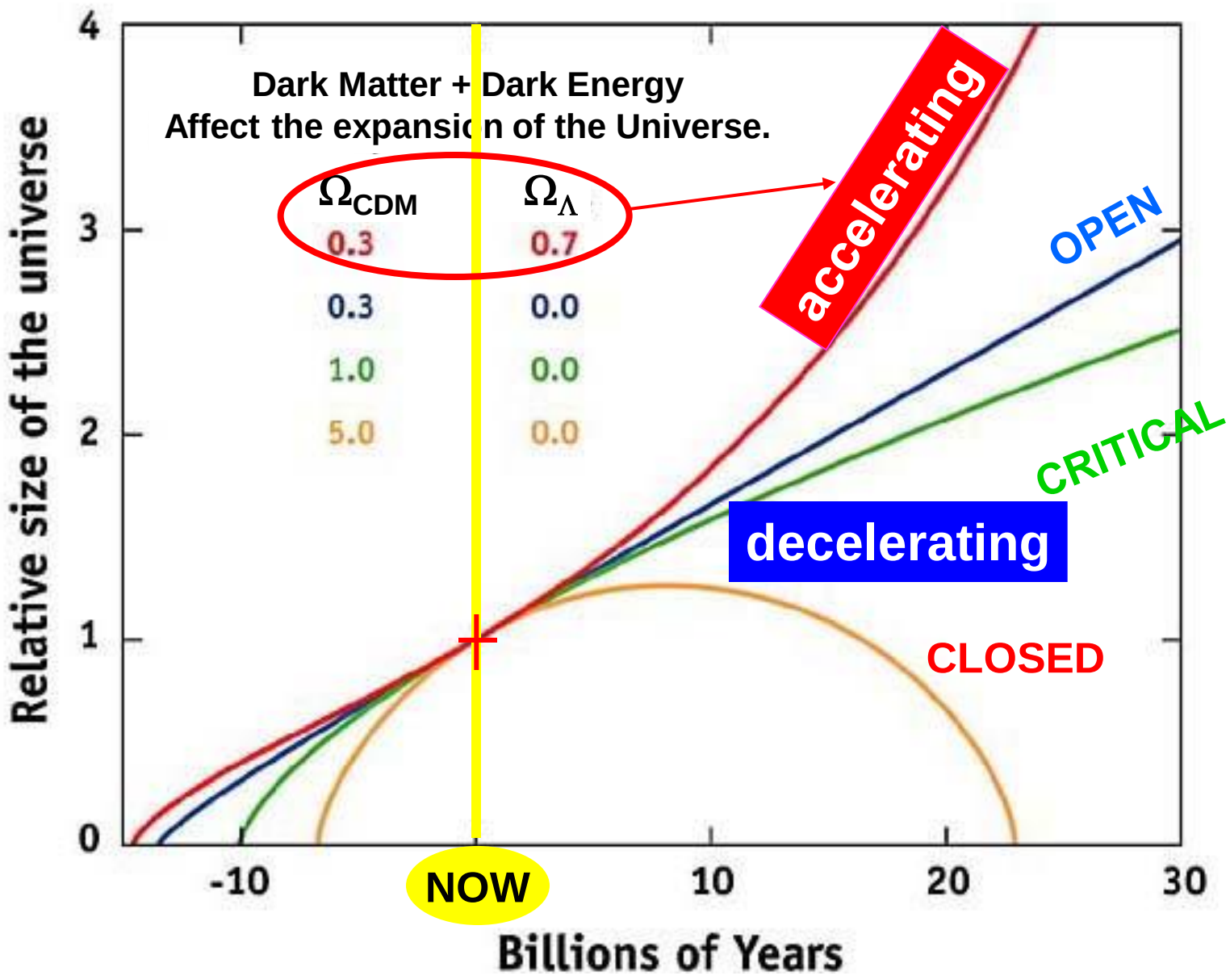
1) Calculate the acceleration, d^2a/dt^2, from the Friedmann equation.

Answer the following questions:

2) Show the sign of d^2a/dt^2 when $\Lambda = 0$.
Cosmic expansion velocity could be zero ($da/dt = 0$), however acceleration d^2a/dt^2 cannot be zero. We cannot live in a stationary and static universe when $\Lambda = 0$.

3) When $\Lambda > 0$, show the condition that holds among cosmological parameters to satisfy $da/dt = 0$ and $d^2a/dt^2 = 0$. We can live in a stationary and static universe only when $\Lambda > 0$.

Cosmic Expansion



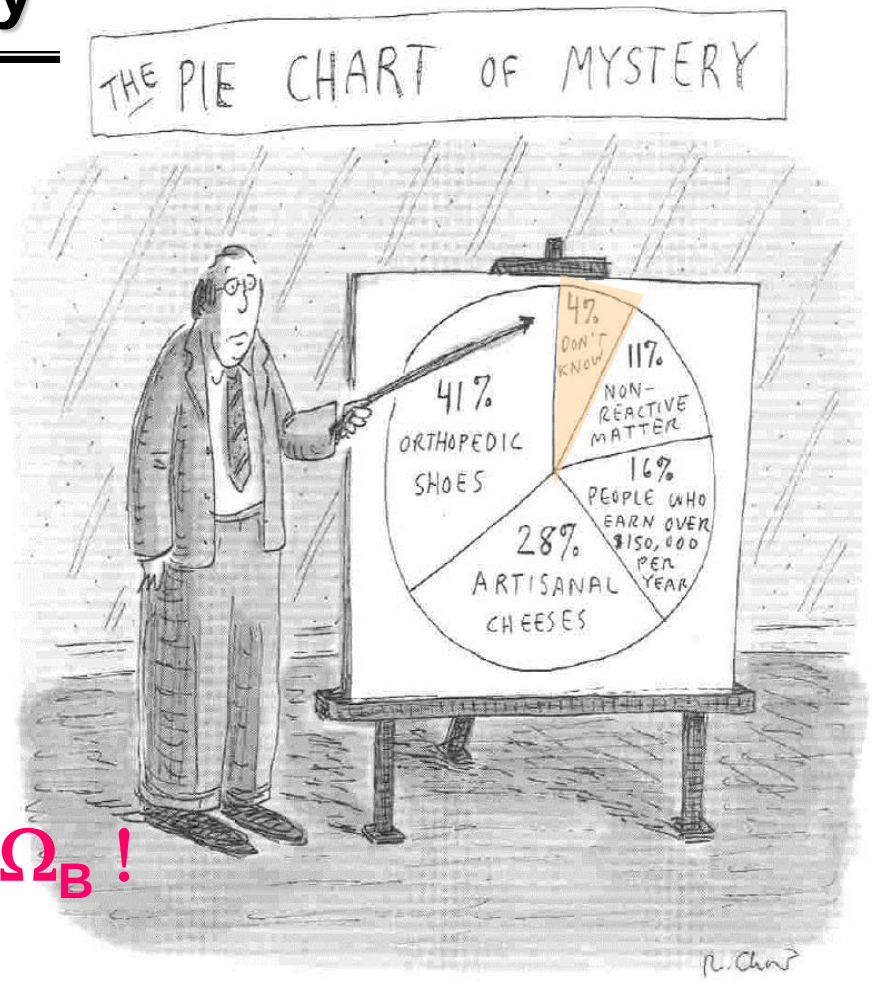
Pie Chart of Cosmic Mystery

Ordinary matter makes up a small fraction of mass/energy.

Dark matter and dark energy dominate.

Ω_Λ ?

Ω_B !



What is the dark component of the Universe!

(HDM: $\Omega_\nu > 0.003$) Ω_{CDM} ?

How to determine the cosmological parameters $\Omega_i=\rho_i/\rho_c$?

- $\Omega_{\gamma\nu} < 0.01\%$ $\Omega_\gamma \rightarrow$ Temperature of CBR (Cosmic Background Radiation)
  $\Omega_\nu \rightarrow$ Upper limit from neutrino oscillation
- $\Omega_\Lambda = 73\%$ Ia Supernovae CMB (Cosmic Background Anisotropies)
- $\Omega_{CDM} = 23\%$ Ia Supernovae CMB (Cosmic Background Anisotropies)
  Gravitaional Lensing
- $\Omega_B = 4\%$ CMB (Cosmic Background Anisotropies)
  Big-Bang Nucleosynthesis

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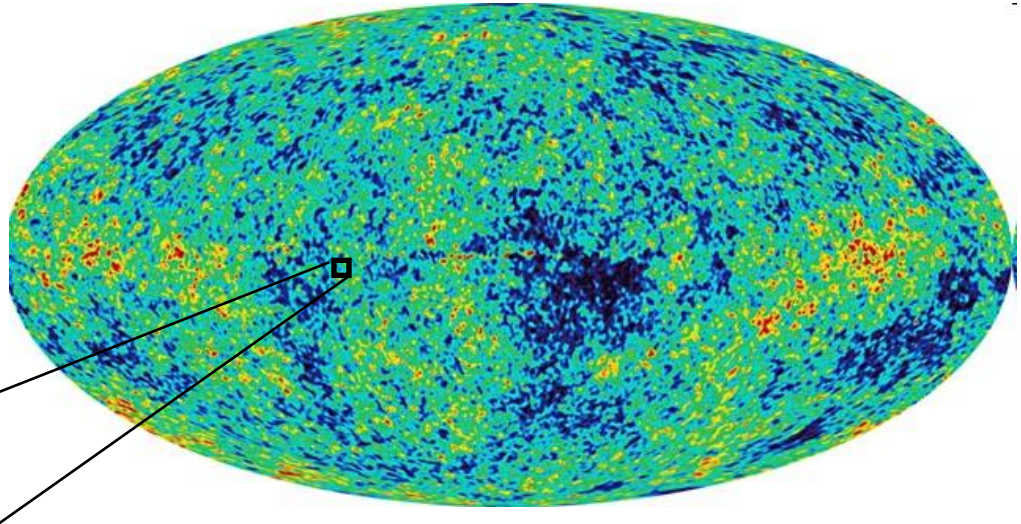
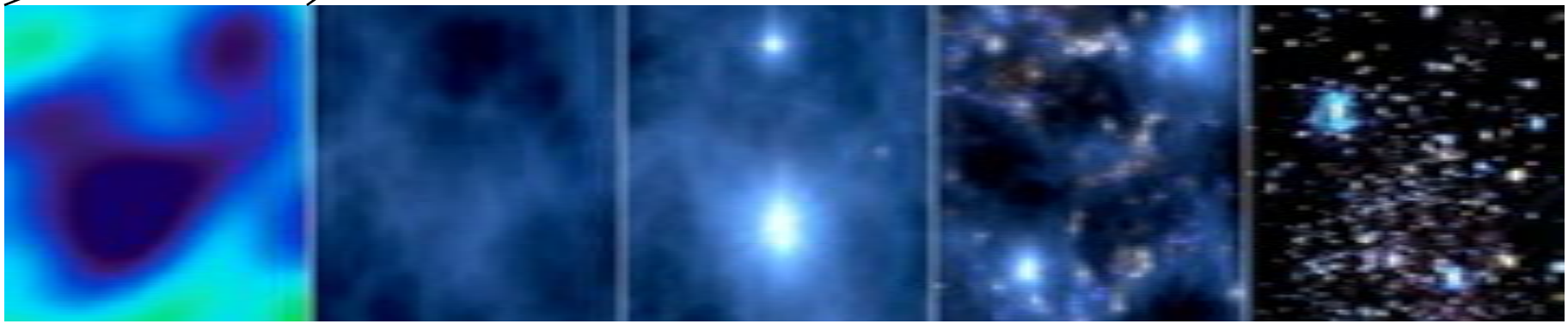
* $\Omega_{\gamma\nu}+\Omega_{CDM}+\Omega_B+\Omega_\Lambda=1$ From all above combination
* Cosmic Age Ia Supernovae + All above combination = 13.7Gy

Primordial fluctuations in hot Big-Bang Universe was discovered by Smoot and Mathar in 1992.



(COBE Satellite)

$t = 3.8 \times 10^5$ y



3.8×10^5 y → 13.7 Gy

Assuming 73% dark energy (DE) and 23% dark matter (DM), computer simulation of cosmic structure formation best explains the observed structure! **What is the nature and origin of DE and DM ?**

Inflationary Hypothesis

Inflationary Scenario

Horizon Problem

$(\delta T/T)_{RECOM.} \approx 10^{-5}$ (Dipole), 10^{-6} (Quadrupole)

EXTREMELY ISOTROPIC ! WHY?

Flatness Problem

$$\frac{k}{R_0^2} = H_0^2\left(\frac{3}{2}\Omega_0 - 1\right) + \left(\frac{\ddot{R}}{R}\right)_0 \longrightarrow 0$$

FLAT ! WHY?

Const. $\mathcal{O}(0)$ TOO SMALL TO BE OBSERVED.

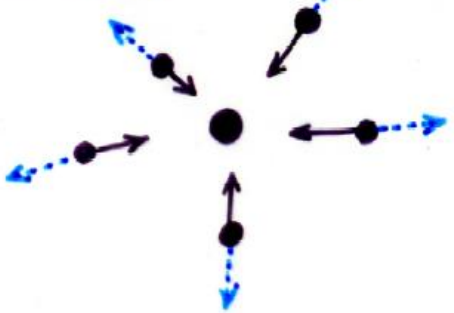
Hubble (Causal) Horizon

$$cH^{-1} \equiv ct_H \equiv c\left(\frac{\dot{R}}{R}\right)^{-1}$$

Hubble time

$$t_c = \frac{1}{n\sigma v}$$

Collision time



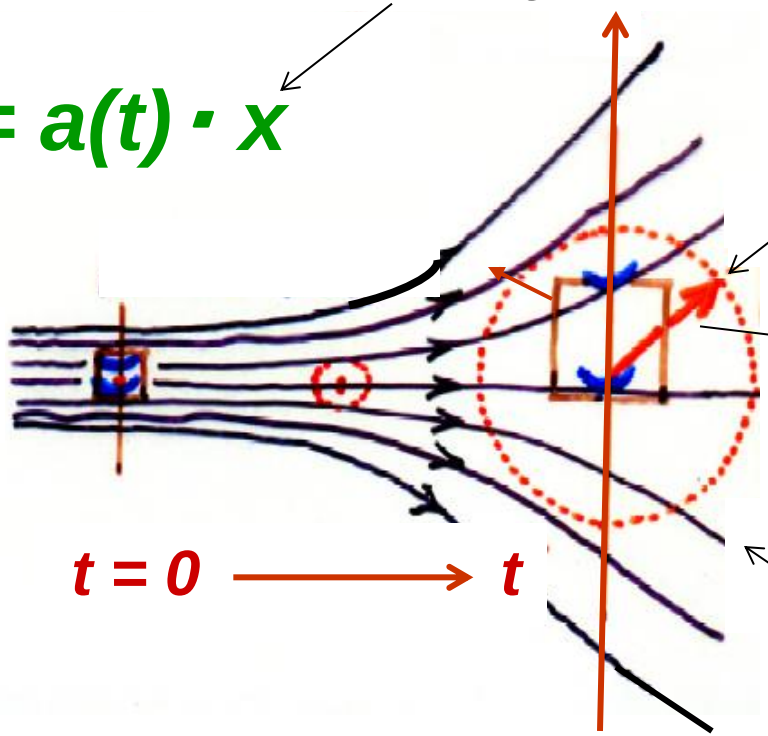
Only when $t_c \lesssim t_H$,

physical process happens CAUSALLY!

Physical Distance

Comoving Coordinate

$d = a(t) \cdot x$



(Causal) Hubble Horizon

Lines of flow in the RIVER

c = Velocity of light is constant in any time t & any scale factor a(t).

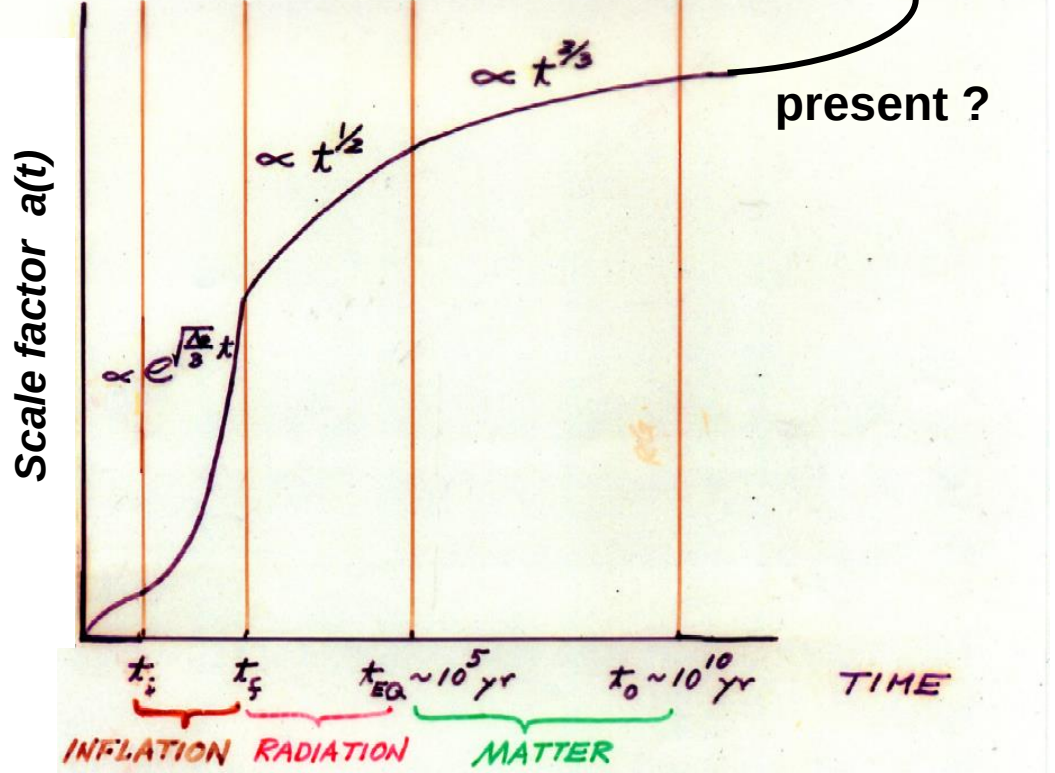
$t = 0 \longrightarrow t$

a(t) = Scale Factor

Epoch	*Scale factor* $a(t)$	*Hubble horizon* $cH^{-1}(t)$
Vacuum-dominated (INFLATION)	$e^{\sqrt{\frac{\Lambda}{3}}t}$	const.
Radiation-dominated	$t^{1/2}$	$2t$
Matter-dominated	$t^{2/3}$	$\frac{3}{2}t$

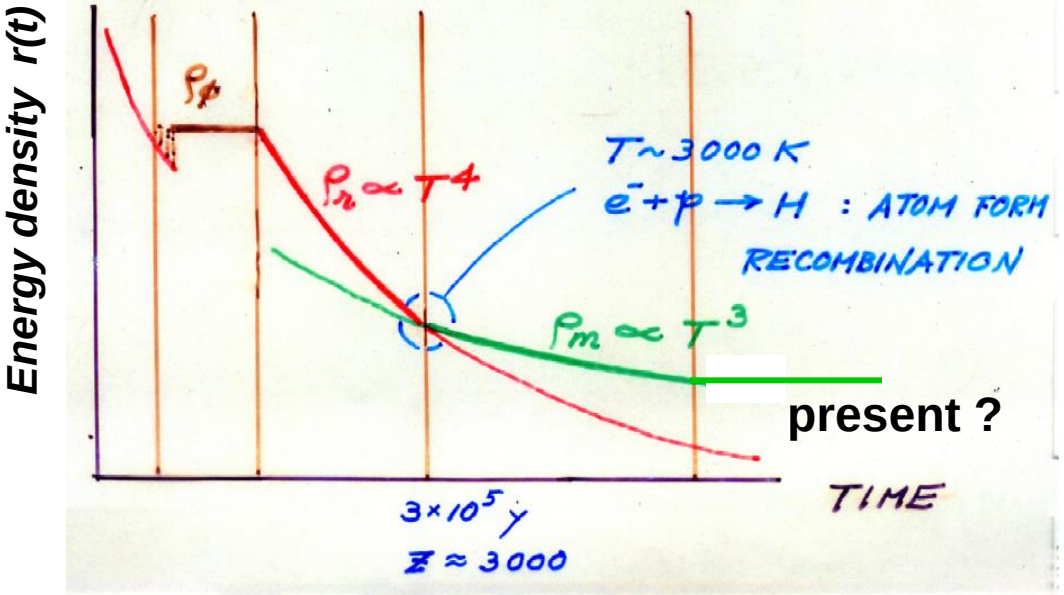
Metric Expansion

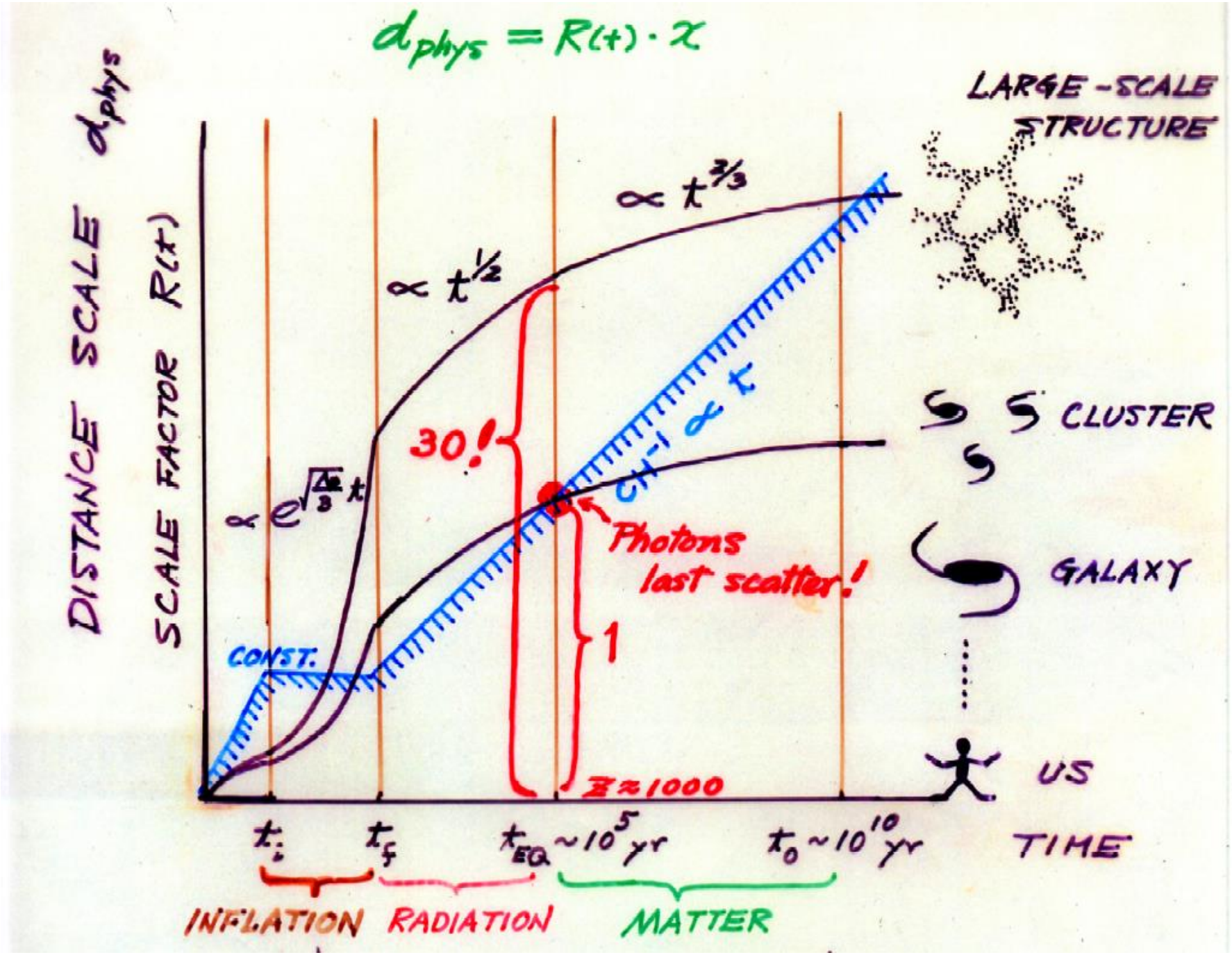
$a(t) \sim \exp(\alpha t)$ (Inf.)
$\sim t^{1/2}$ (RD)
$\sim t^{2/3}$ (MD)
$\sim \exp(\beta t)$ (present)

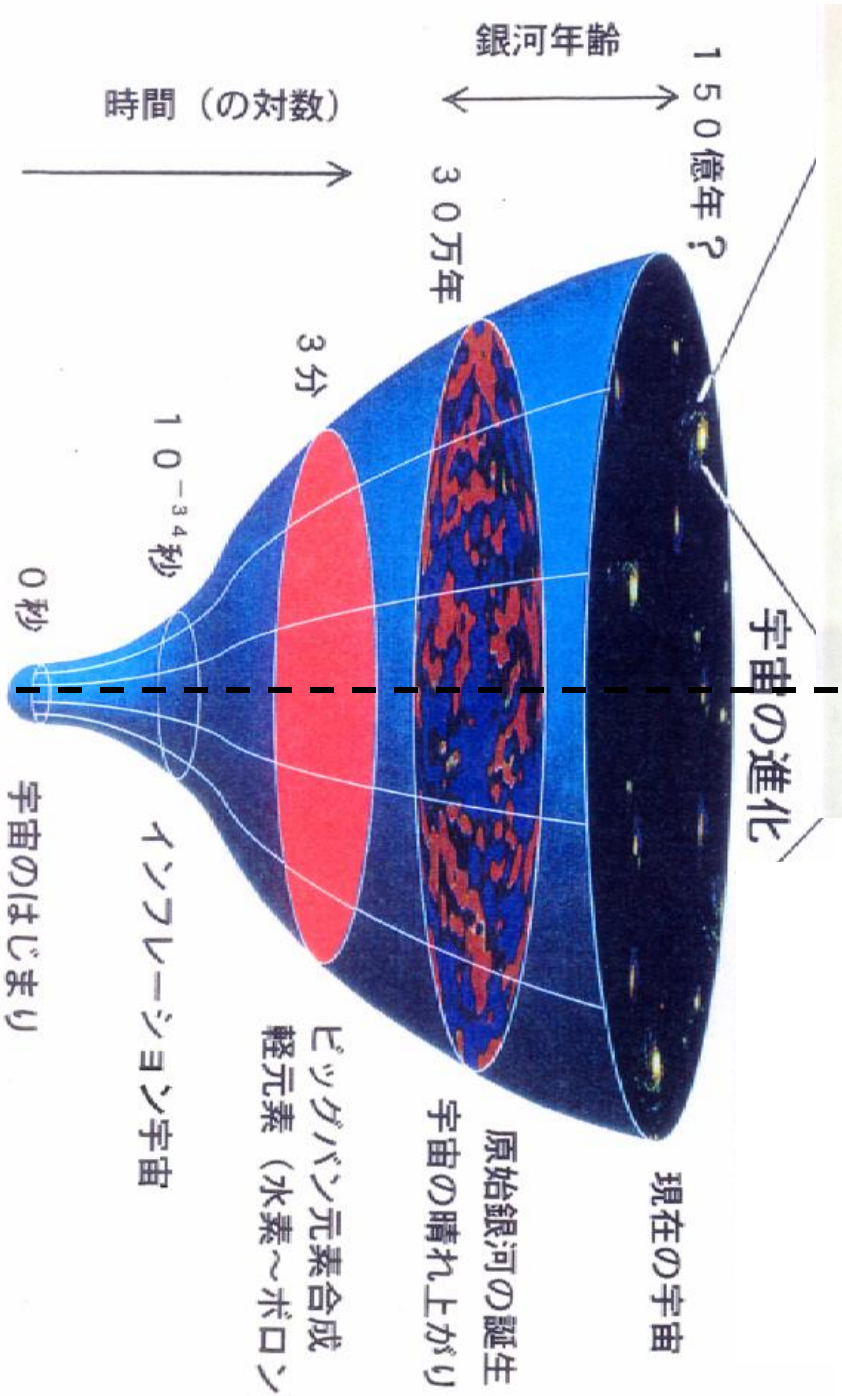


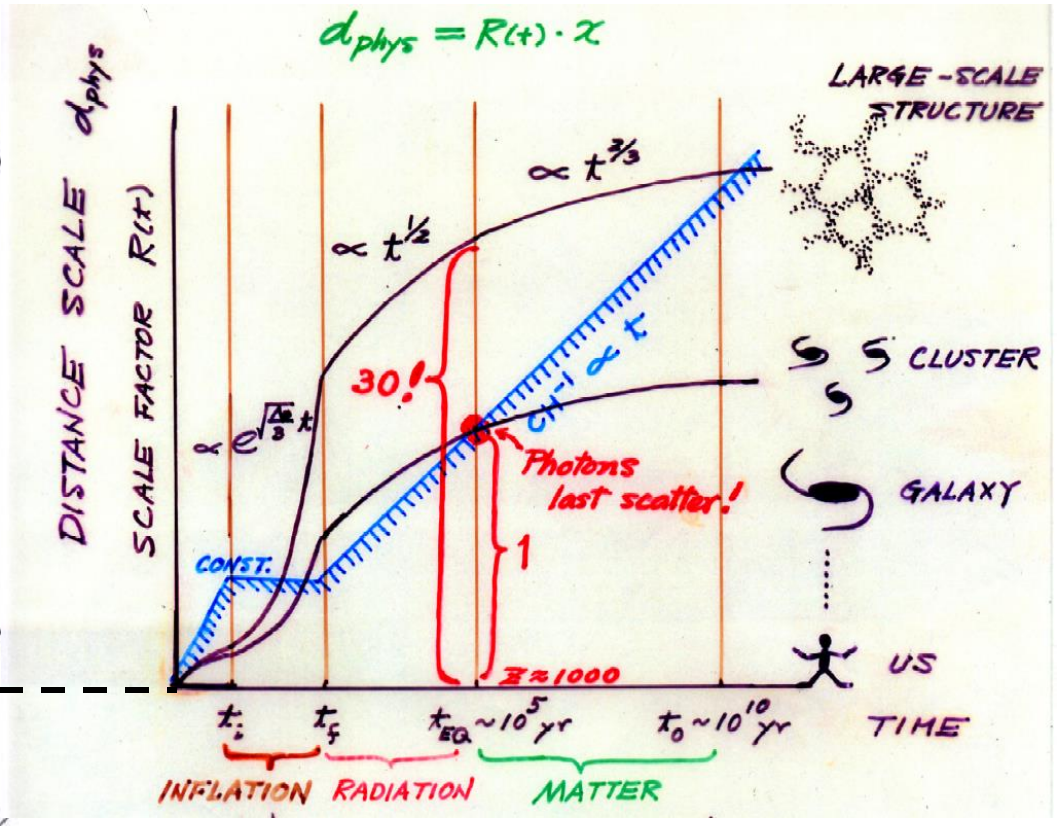
Energy Density

$\rho(t) \sim$ const. (Inf.)
$\sim T^4$ (RD)
$\sim T^3$ (MD)
$\sim$ const (present)




$d_{phys} = R(t) \cdot x$
DISTANCE SCALE d_{phys}
SCALE FACTOR $R(t)$
LARGE-SCALE STRUCTURE
$\propto t^{2/3}$
$\propto t^{1/2}$
$\propto e^{\sqrt{\frac{\Lambda}{3}}t}$
$ct^{-1} \propto t$
30!
Photons last scatter!
1
$z \approx 1000$
CONST.
CLUSTER
GALAXY
US
t_i
t_f
t_{EQ}
$\sim 10^5$ yr
$t_0 \sim 10^{10}$ yr
TIME
INFLATION
RADIATION
MATTER





- **Causal horizon expands at the speed of light.**
- **Space extends even beyond the causal horizon.**

- **Good bye (Inflation) → Hello (RD/MD) - SCENARIO**

DIS-CONNEXTED → **HORIZON-in**

Quantum Fluctuation
Causally Connected

HORIZON PROBLEM, SOLVED !

- **Exponential Expansion of CURVED-SPACE**

INFLATION at GUTs EPOCH

$t_i \approx 10^{-34}$ sec → $t_f \approx 10^{-32}$ sec

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 \approx \underbrace{\left(\frac{8\pi G}{3}\rho_\phi + \frac{\Lambda}{3}\right)}_{\Lambda_\phi/3} - \frac{k}{R^2}$$

$$H_i = \frac{1}{2t_i} \sim 10^{+34} \approx \sqrt{\frac{\Lambda_\phi}{3}}$$

$$\left|\frac{k}{R_f^2}\right| = \left|\frac{k}{R_i^2}\right| \times \left(\frac{R_i}{R_f}\right)^2 = \left|\frac{k}{R_i^2}\right| \exp\left[2\sqrt{\frac{\Lambda_\phi}{3}}(t_i - t_f)\right] \sim \left|\frac{k}{R_i^2}\right| \times 10^{-87}$$

FLATNESS PROBLEM, SOLVED !

$$1 = \Omega_M + \Omega_\Lambda$$