Radiative Processes in Astrophysics I / Stellar Physics, Advanced Course IV
AY2016 End-of-Class Report

平成28年度(2016年度)冬学期
天体輻射論I／恒星物理学特論IV 期末レポート課題

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◆**Undergraduate students (B4,B3,B2):** Choose Report 1.

◆**Graduate students (M2,M1):** Choose either Report 1 or 2.

学部生はReport1を、大学院生はReport1か2のどちらかを選択せよ。
（なおレポートは日本語でよい）

Report 1

◇B4, M2 (修了生): Choose two problems.
◇M1, B3, B2 (それ以外): Choose three problems.
Then, submit the answers/discussions with A4-papers (no-limit on the number of pages).

<u>Problem1</u> *Two-stream approximation* 2流近似

1) Show that the hemi-isotropic radiation field (see eq.(3.10) in your note):

$$I(\tau;\mu) \equiv \begin{cases} I^+(\tau) & (0 \le \mu \le 1) \\ I^-(\tau) & (-1 \le \mu \le 0) \end{cases} \tag{1}$$

satisfies the Eddington approximation ($K = \frac{1}{3}J$).
2) As an extreme of the above hemi-isotropic case, the radiation field can be represented by the following two-beams in the opposite directions:

$$I(\tau;\mu) \equiv \begin{cases} I^+(\tau)\,\delta(\mu-\mu_0) & (0 \le \mu \le 1) \\ I^-(\tau)\,\delta(\mu+\mu_0) & (-1 \le \mu \le 0) \end{cases} \tag{2}$$

where μ_0 is a constant ($0 \le \mu_0 \le 1$). Show that this radiation field also satisfies the Eddington approximation, but only in case $\mu_0 = \frac{1}{\sqrt{3}}$. Then, discuss briefly why this specific direction can represent the semi-isotropic radiation field that satisfies the Eddington approximation.

<u>Problem2</u> *Iterated Eddington approximation* 反復エディントン近似

With the Eddington approximation ($K = \frac{1}{3}J$) for the gray stellar atmosphere, we obtained the following solution of linear atmosphere (eq 3.28):

$$S(\tau) = J(\tau) = 3H_0(\tau + \frac{2}{3}), \tag{3}$$

where S(τ) is the source function of the atmosphere as as function of the optical depth (τ) measured from the stellar surface, $J(\tau)$ is the 0th moment of the radiation field, and H_0 is the 1st moment of the outgoing radiation field and it is a constant.
By putting this into the formula (eq 3.21), we got the formal solution:

$$I(\tau;\mu) = 3H_0(\tau + \mu + \frac{2}{3}) \tag{4}$$

for $0 \leq \mu \leq 1$ (eq 3.29).

1) Similarly obtain the formal solution $I(\tau;\mu)$ for negative μ ($-1 \leq \mu \leq 0$). Note the integration range for τ' is $0 \to \tau$ as compared to $\tau \to \infty$ for positive μ.
2) Using $I(\tau;\mu)$ for both positive and negative μ, calculate the 1st moment of radiation, thus the source function. Show they are:

$$S^{(1)}(\tau) = J^{(1)}(\tau) \equiv 3H_0\{\tau + \frac{2}{3} - \frac{1}{3}\,E_2(\tau) + \frac{1}{2}\,E_3(\tau)\}, \tag{5}$$

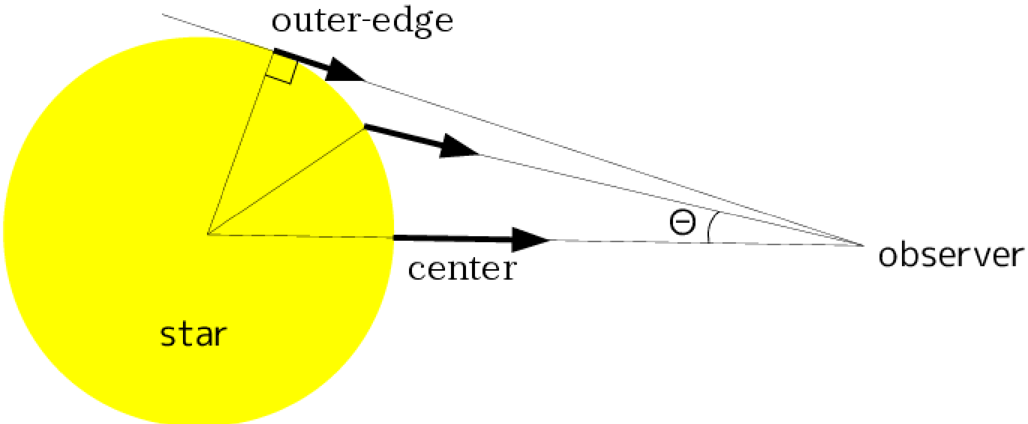
where E_n is called *exponential integral* defined as:

$$E_n(x) \equiv \int_1^\infty e^{-xt}\, t^{-n}\, dt \tag{6}$$

Note the newly obtained $S^{(1)}$ differs slightly from the initial S in Eq.3. $S^{(1)}$ can be regarded as 1st order approximation of this iteration and you might reach to the exact solution of the radiation field by continuing this iteration (this process is called "iterated Eddington approximation").

Problem3 *Limb darkening* 周縁減光

If you take a picture of a star at a close distance, you can obtain the two-dimensional image of the stellar disk. Interestingly surface brightness of stellar disk is not uniform across the disk, but gradually decreases from the center to the edge. This is called "limb darkening", which occurs because the radiation from the stellar surface is NOT a Lambertian, which has a uniform intensity for all directions.



1) Show the brightness profile normalized with the central brightness (I_c) for the classical stellar atmospheric model (Eq.4) is

$$\frac{I(\Theta)}{I_c} = \frac{2}{5} + \frac{3}{5}\sqrt{1 - (\frac{\mathrm{D}}{\mathrm{R}_\odot})^2 sin^2(\Theta)}, \tag{7}$$

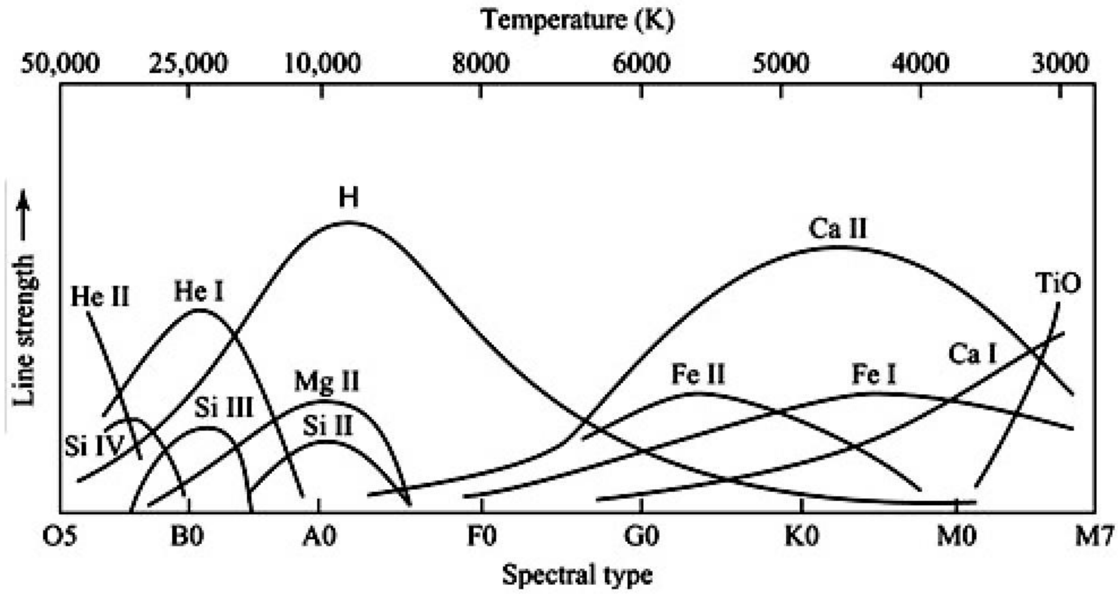
where $\mathrm{R}_\odot$ is the radius of the star, and D is the distance to the star, and Θ is the angular distance from the center of the star (see the above figure).
2) Show that the emergent radiation field of the 1st-iterated Eddington approximation is:

$$I^{(1)}(0;\mu) = 3H_0\{\frac{7}{12} + \frac{1}{2}\mu + (\frac{1}{3}\mu + \frac{1}{2}\mu^2)\,\ln\frac{1+\mu}{\mu}\}. \tag{8}$$

Then, derive the brightness profile.
3) Plot both brightness profiles for the above two Eddington approximations (0th and 1st) to compare them, then discuss the difference.

Problem4 *Saha equation* サハの式

In § 4.4, we learned why hydrogen absorption line is strongest in A-type stars with $T_{eff} \sim 10000$ K. Following the same argument, discuss the reason why FeII lines are strongest at $T_{eff} \sim$ 5000-6000 K while FeI lines are strongest at $T_{eff} \sim$ 4000-5000 K. See any books or internet resources for the necessary numbers.



Report 2

In this course, a variety of radiative processes in astrophysics have been introduced with some applications to real astronomical objects. Choose one of the following two assignments to submit the report with A4-papers (no-limit on the number of pages).

講義では、紫外線・可視光・赤外線における主な天体輻射過程が、幾つかの応用例とともに紹介された。以下の 2 つの課題から**1つを選び**、A4 レポート用紙にまとめよ（枚数は問わない）。

【Assignment A】 Consider and describe any possiblity of new knowledge/finding/insight by applying any radiative processes in astrophysics that you learned in this course to your currently on-going research or any research you hope to do next.

【課題 A】自分が現在進めている (もしくは進めようとしている) 研究テーマについて、今回学んだ天体輻射過程や関連した物理を用いて**何か新しい知見が得られないか**、自由に考察して記述せよ（新しい知見に対する言及を必ず含めること）。

【Assignment B】 Pick up a radiative process which you intuitively become most intrigued or concerned. Study the process with books/web pages until you are satisfied to summarize in a report.

【課題 B】直感的に最も興味を持った天体輻射過程、もしくは講義中に疑問に思い**どうしても気になった**項目があれば、それについて文献等で詳細を調べ、納得するまで考察し、その内容を記述せよ。

Evaluation 評価について

- This end-of-class report will be evaluated by the depth of your consideration/discussion/research. Typos or small mistakes do not matter that much, but the logic of the discussion will be evaluated as one of the most important points. Better use figures/tables as much as possible.

  本期末レポートは、どれだけ深く考えているか、ないし、どれだけよく調べているかを判断して評価する。計算や式変形などの細かい間違いは問わないが、記述が論理的か・わかりやすいかは評価判断材料とする。図表を活用するのが好ましい。

- Those who finish the school at the end of this academic year, please contact me in case you want to check if you can really get enough number of credits.

  修了生で単位取得を早急に確認したい人は、別途小林まで連絡をするように。

Note 注意事項

(1) Use only one side of A4-paper and put page numbers on all pages. Do not staple the papers but bind them with a clip etc. 記述は片面とし、ページ番号を必ずうっておくこと。また、ホッチキスではなくクリップ等はずしやすいものでとめておくこと。

(2) If you use any references or internet resources, refer them in the report. If you discuss with your friends, acknowledge the opportunity in the report. 使用した参考書があれば、書名を書いておくこと。また議論した友人がいれば、その友人の名前をあげ感謝の意を表すること。

(3) レポートには忘れずに学生証番号、氏名を記入すること。

Submit to 提出先

Mr.Onozuka at the office of the Astronomy Department

天文教室事務 小野塚さんまで

Deadline 提出締め切り

M2(修了生): 2017/2/10(Fri) 17:00

B4(修了生): 2017/2/10(Fri) 17:00

M1,B3,B2: 2017/2/28(Tue) 17:00