Exoplanets III.
Outline

- Fundamentals of Stellar Physics
- Asteroseismology
- Finding of Invisible Companions in Binary Systems

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I. Fundamentals of Stellar Physics

Observational Facts

Characteristic quantities

- Size $R_{sun} = 7\ 10^8$ m
- Mass $M_{sun} = 2\ 10^{30}$ kg
- Luminosity $L_{sun} = 4\ 10^{26}$ W
- Dyn timescale $\tau_{dyn} = (GM/R^3)^{-1/2} \sim 1$ hr
- Therm timescale $\tau_{KH} = GM^2/(RL) \sim 10^7$ yr

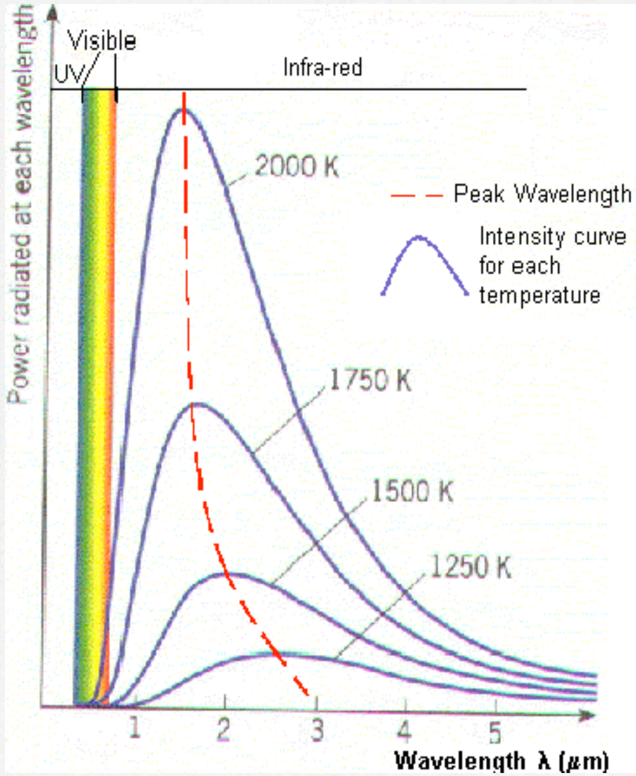
Light from a star

Thermal radiation from a body with *T*

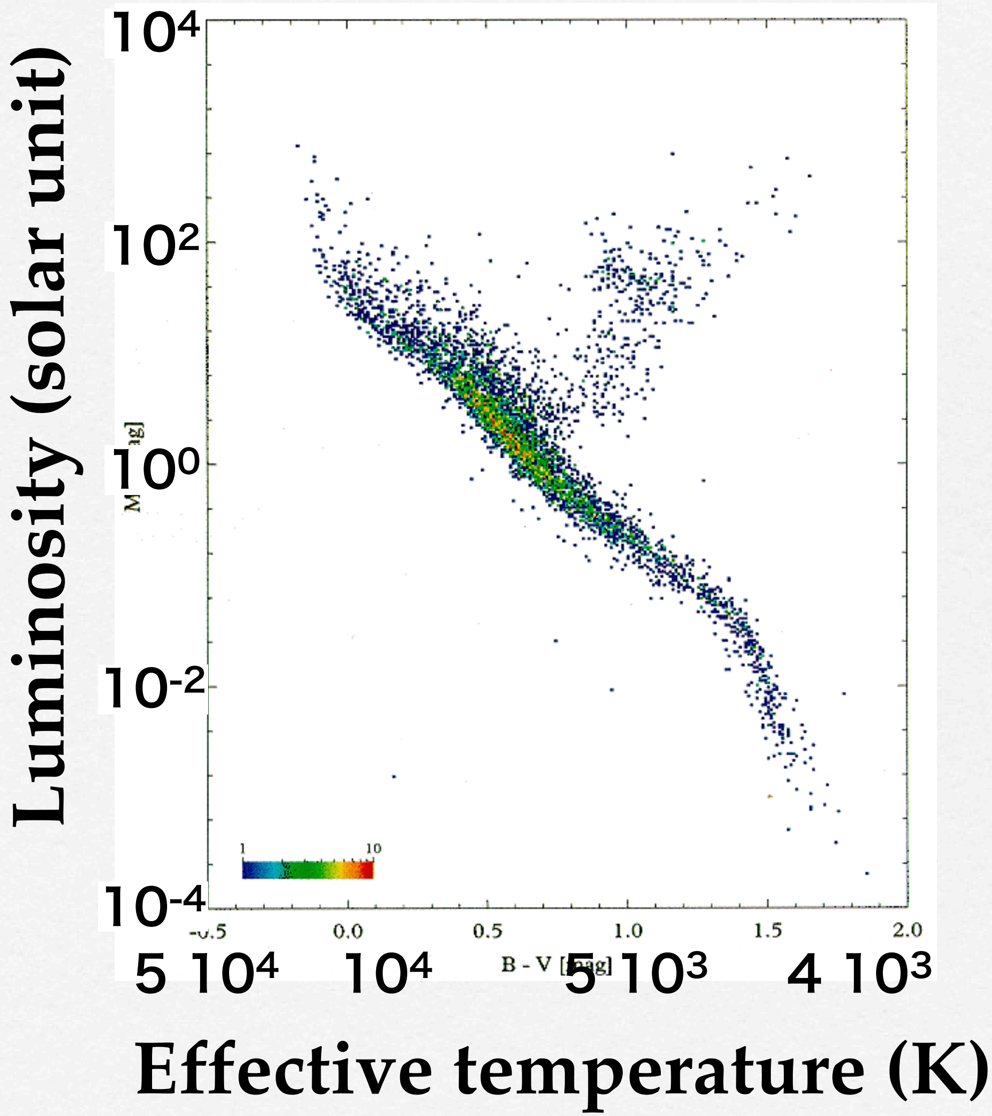
$$B_\lambda(T) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{\exp(hc/\lambda kT)-1}$$

- **Temperature determines spectrum**
- **Colour indicates temperature**

$$\lambda_{max}T = 2.9\ 10^{-3}\ \mathrm{m\ K}$$



Effective Temperature vs Luminosity



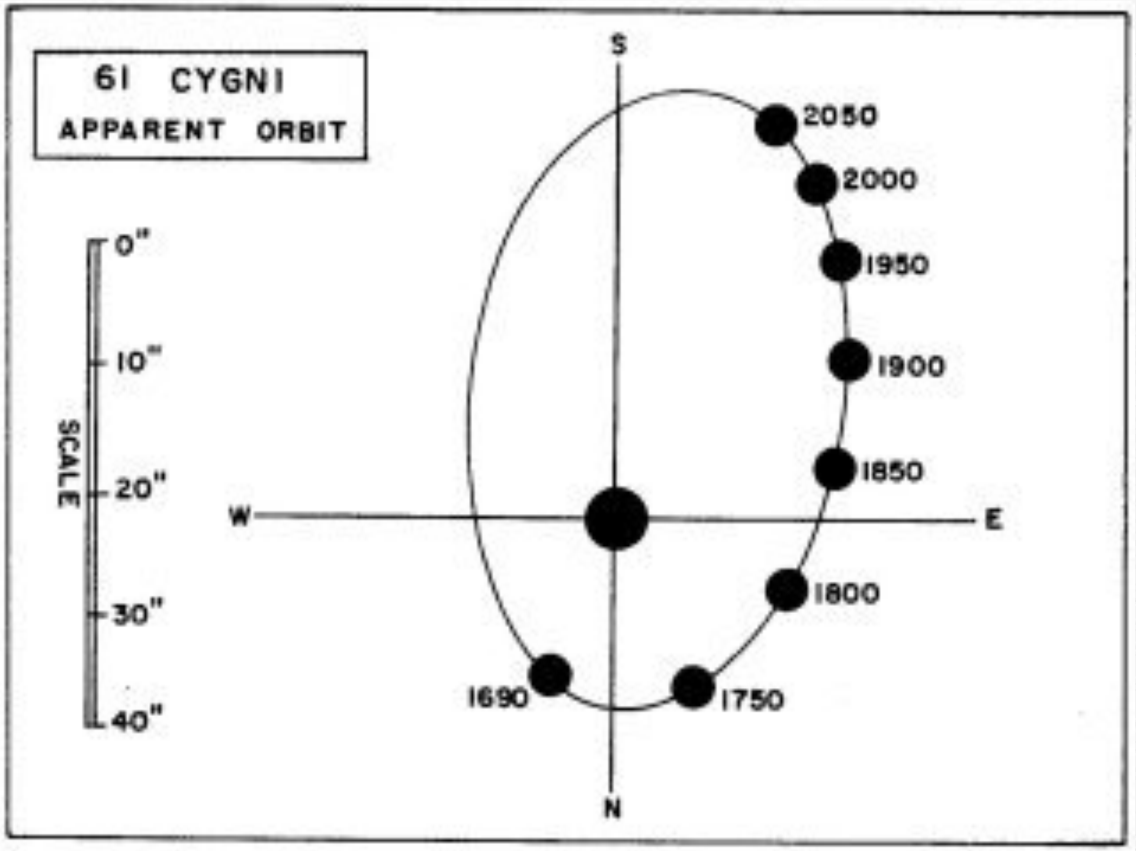
parallax —> distance

apparent brightness + distance —> luminosity

Determination of Stellar Mass

Observables：

(i) period *P*
(ii) parallax *p*
—> distance *d*
(iii) orbit size r_1&r_2

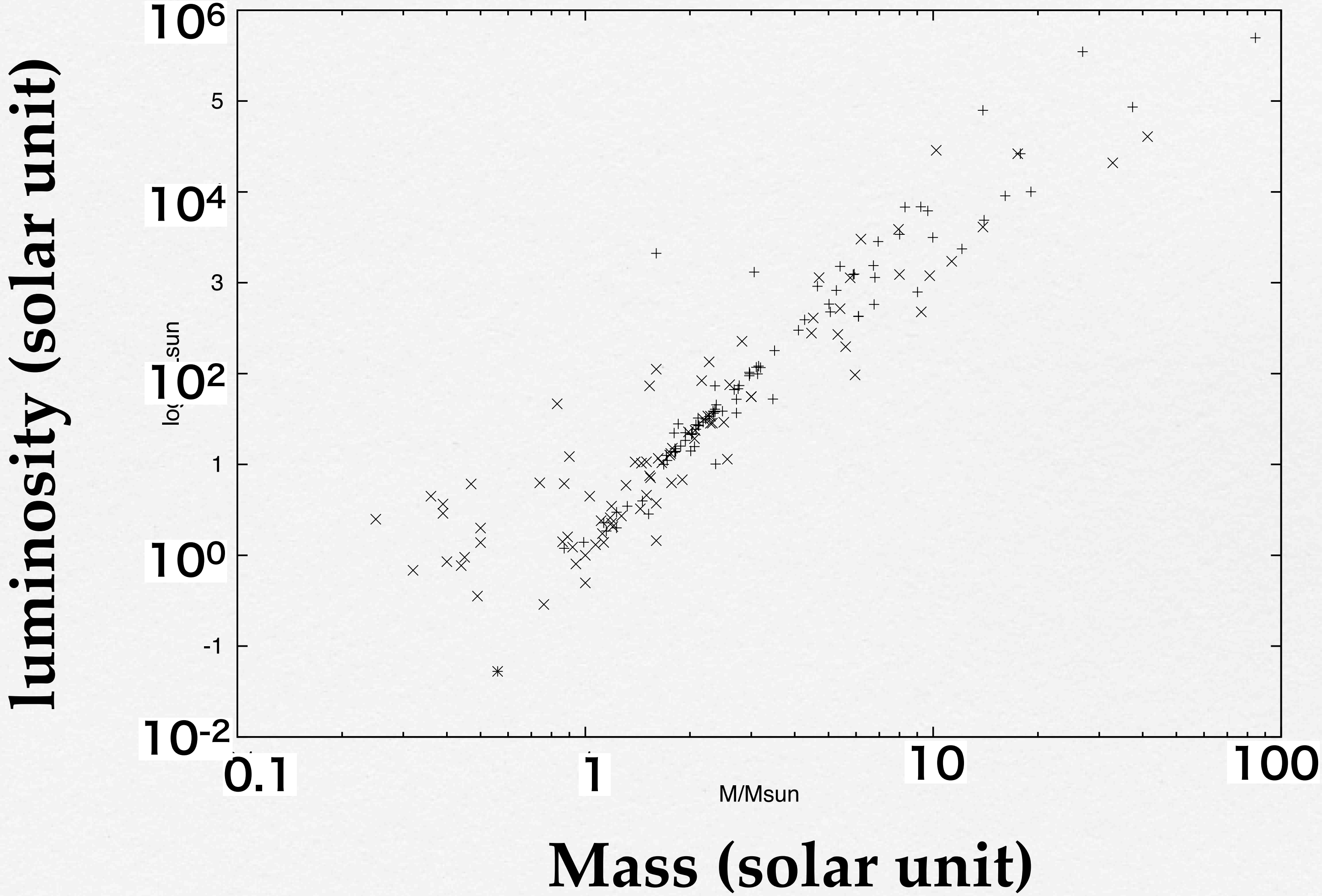


gravity = centrifugal force

$$2Gm_1m_2/(r_1+r_2)^2 = 4\pi^2(m_1r_1+m_2r_2)/P^2$$

$$r_1/r_2 = m_2/m_1$$

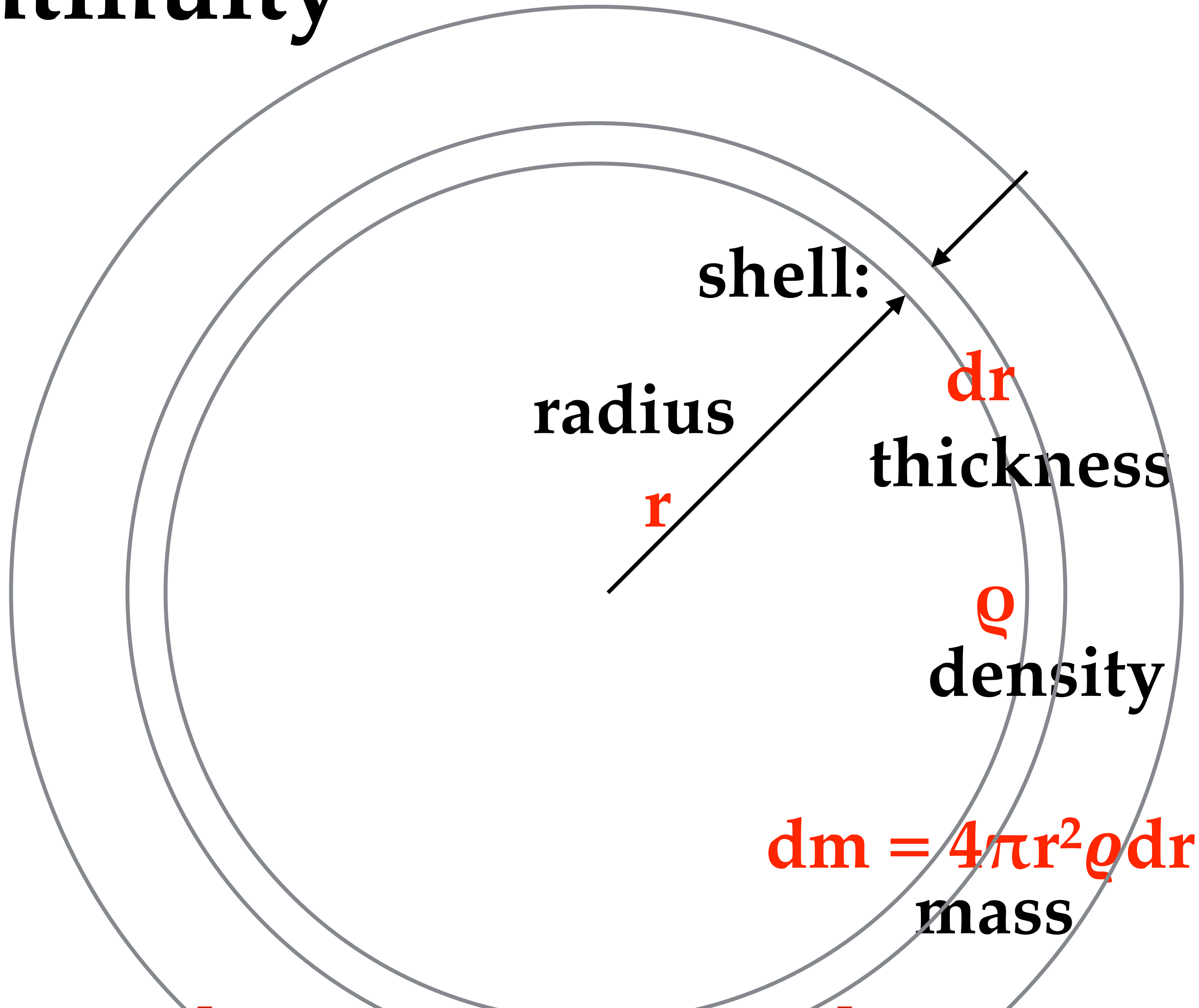
Mass-Luminosity relation



Theoretical consideration

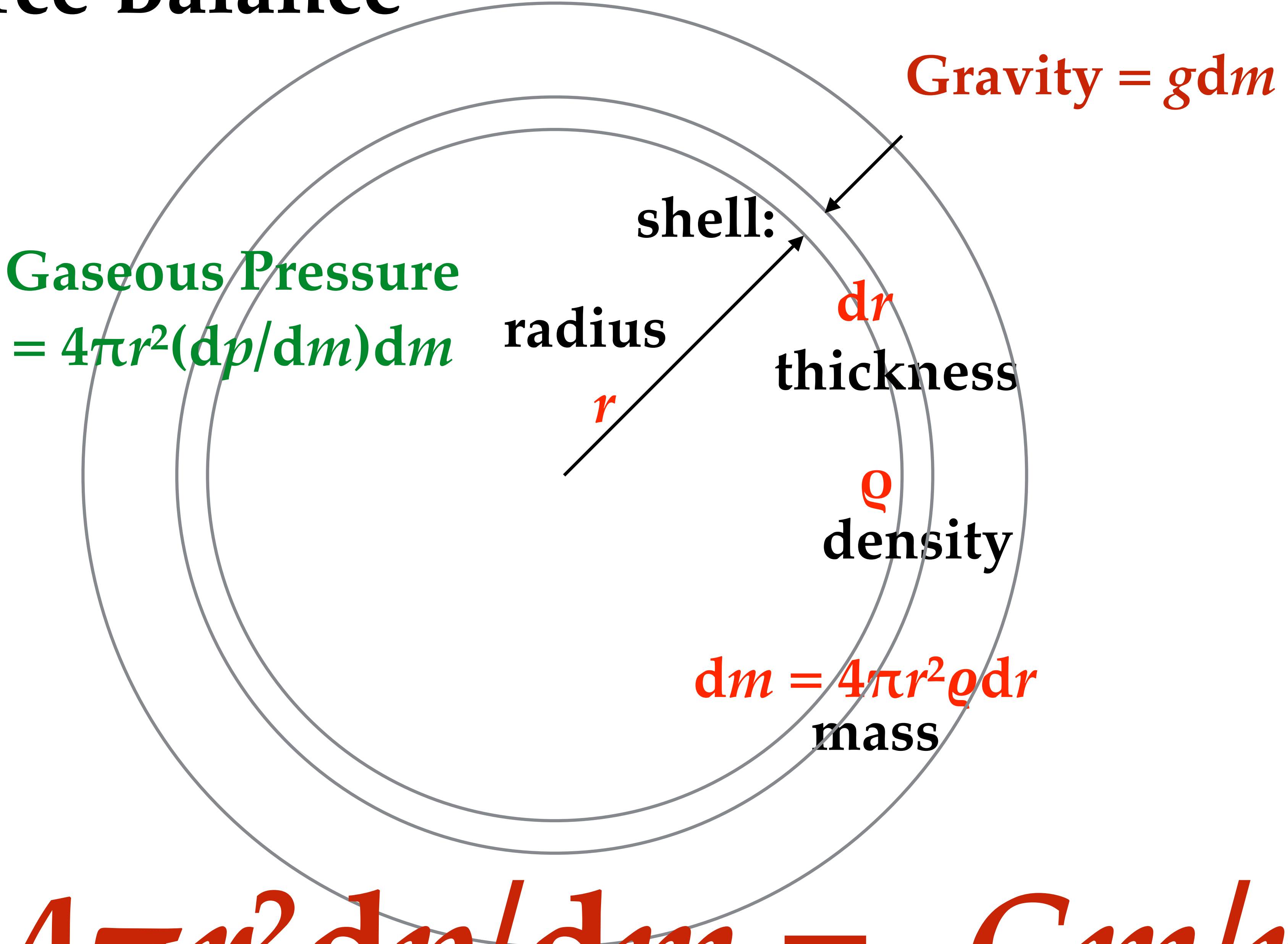
Why are stars shining?

Continuity



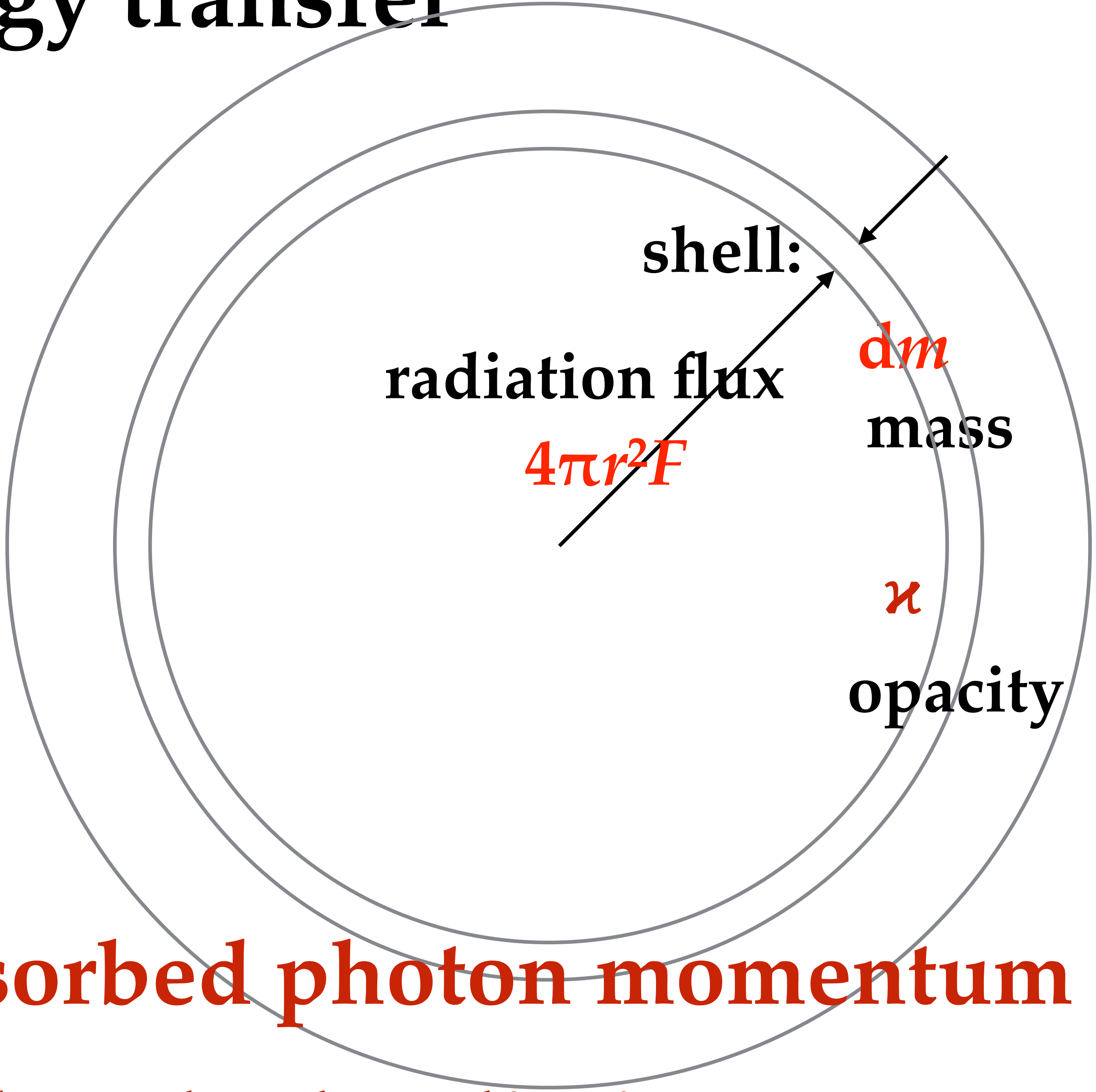
$$dr/dm = 1/(4\pi r^2 \varrho)$$

Force Balance



$$4\pi r^2 \mathrm{d}p/\mathrm{d}m = -Gm/r^2$$

Energy transfer

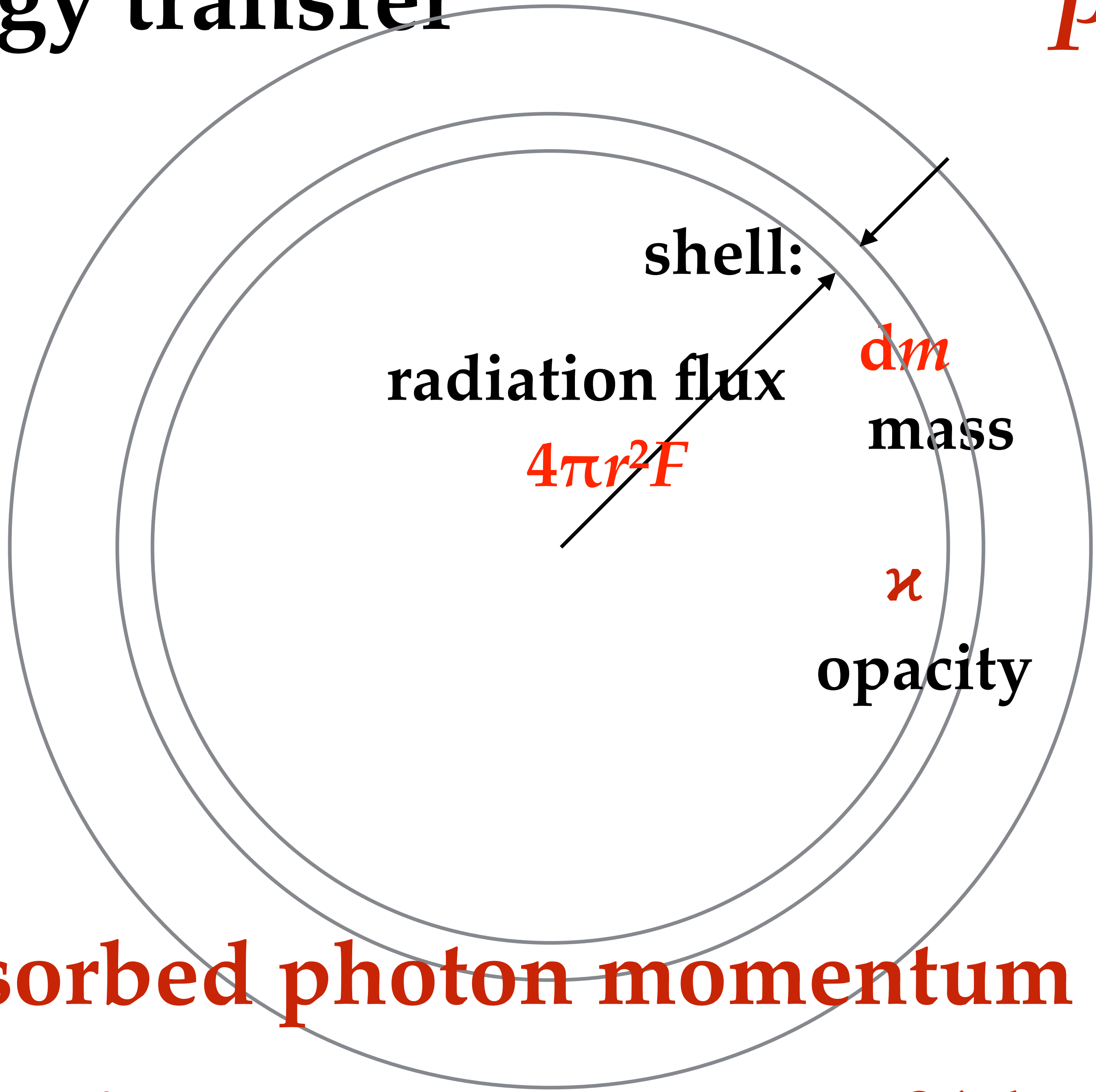


absorbed photon momentum = $F\varkappa \mathrm{d}m/c$

absorbed radiation energy = $F\varkappa \mathrm{d}m$

Energy transfer

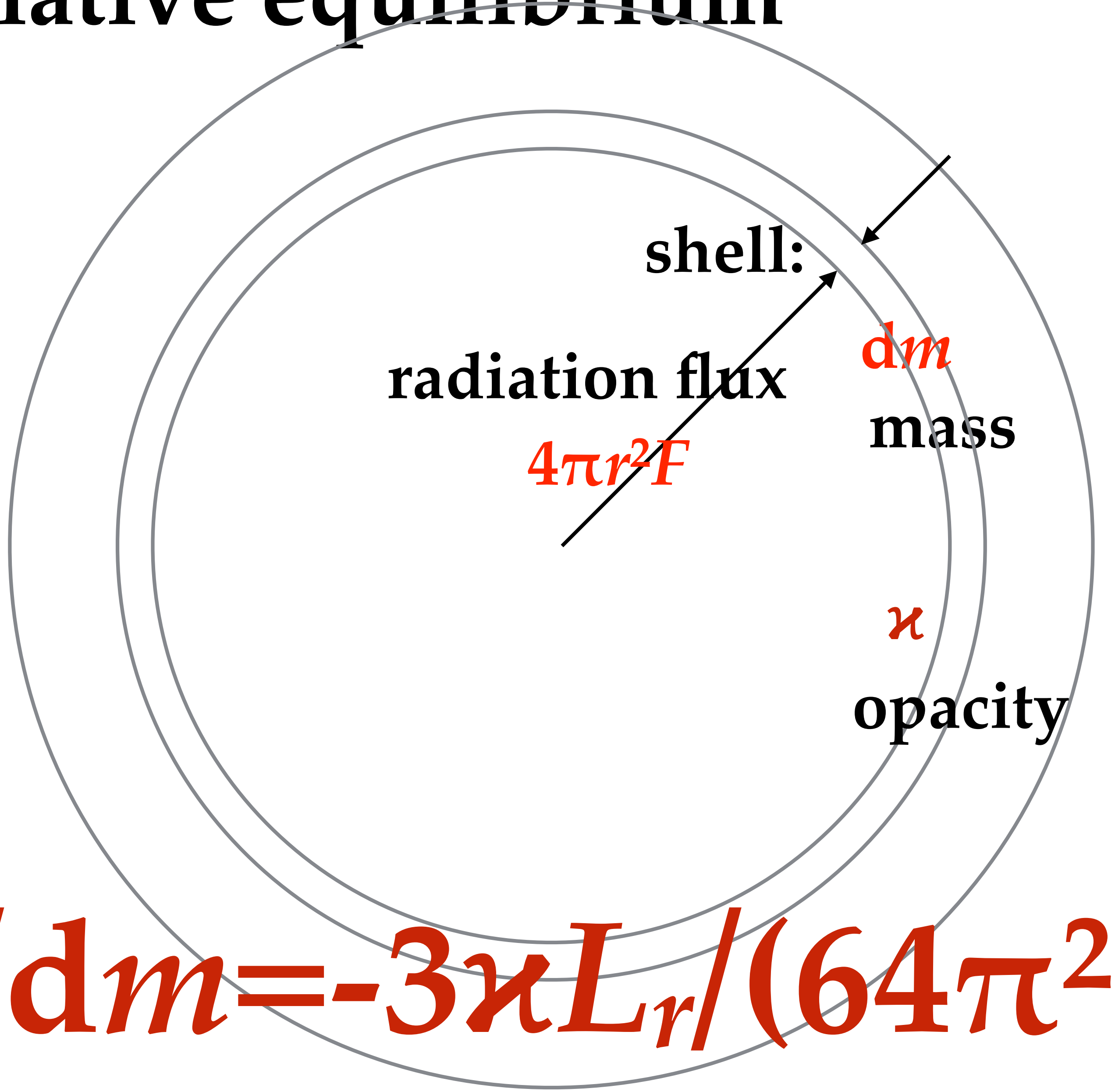
$$P_{\mathrm{rad}} = aT^4/3$$



absorbed photon momentum = $F\varkappa\,\mathrm{d}m/c$

radiation pressure = $-4\pi r^2(\mathrm{d}P_{\mathrm{rad}}/\mathrm{d}m)\,\mathrm{d}m$

Radiative equilibrium



$$dT/dm = -3\varkappa L_r/(64\pi^2 a c r^4 T^3)$$

Equilibrium state

$$\mathrm{d}r/\mathrm{d}m = 1/(4\pi r^2 \rho)$$

$$\mathrm{d}p/\mathrm{d}m = -\, Gm\rho/(4\pi r^4)$$

$$\mathrm{d}T/\mathrm{d}m = -\, 3\kappa L_r/(64\pi^2 acr^4 T^3)$$

Rough estimate:

Differential Eq. --> Difference Eq.

LHS: Difference between Surface and Center

RHS: Averaged values

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Differential Eq. --> Difference Eq.

LHS: Difference between Surface and Center

RHS: Averaged values

$$\mathrm{d}r/\mathrm{d}m = 1/(4\pi r^2\varrho)$$

$$\text{LHS} \approx R/M$$

$$\text{RHS} \approx (4\pi)^{-1}(R/2)^{-2}(\varrho_c/2)^{-1}$$

$$\therefore \varrho_c \approx (2/\pi)(M/R^3)$$

Rough estimate:

Differential Eq. --> Difference Eq.

LHS: Difference between Surface and Center

RHS: Averaged values

$$\mathrm{d}p/\mathrm{d}m = - Gm/(4\pi r^4)$$

$$\text{LHS} \approx -p_c/R$$

$$\text{RHS} \approx -G/(4\pi)\,(M/2)(R/2)^{-4}$$

$$\therefore\ p_c \approx (2/\pi)(GM^2/R^4)$$

The central temperature

Ideal gas

$p = nkT$

$= (\varrho/\mu m_u)kT$

$$\therefore T_c \approx (k/\mu m_u)^{-1}GM/R$$

$\approx 10^7$ K for M_{sun} R_{sun}

n : particle numbers

μ : mean molecular weight

k = Boltzmann constant ($1.38\ 10^{-23}$ J/K)

m_u = atomic weight ($1.66\ 10^{-25}$ kg)

Rough estimate:

Differential Eq. --> Difference Eq.

LHS: Difference between Surface and Center

RHS: Averaged values

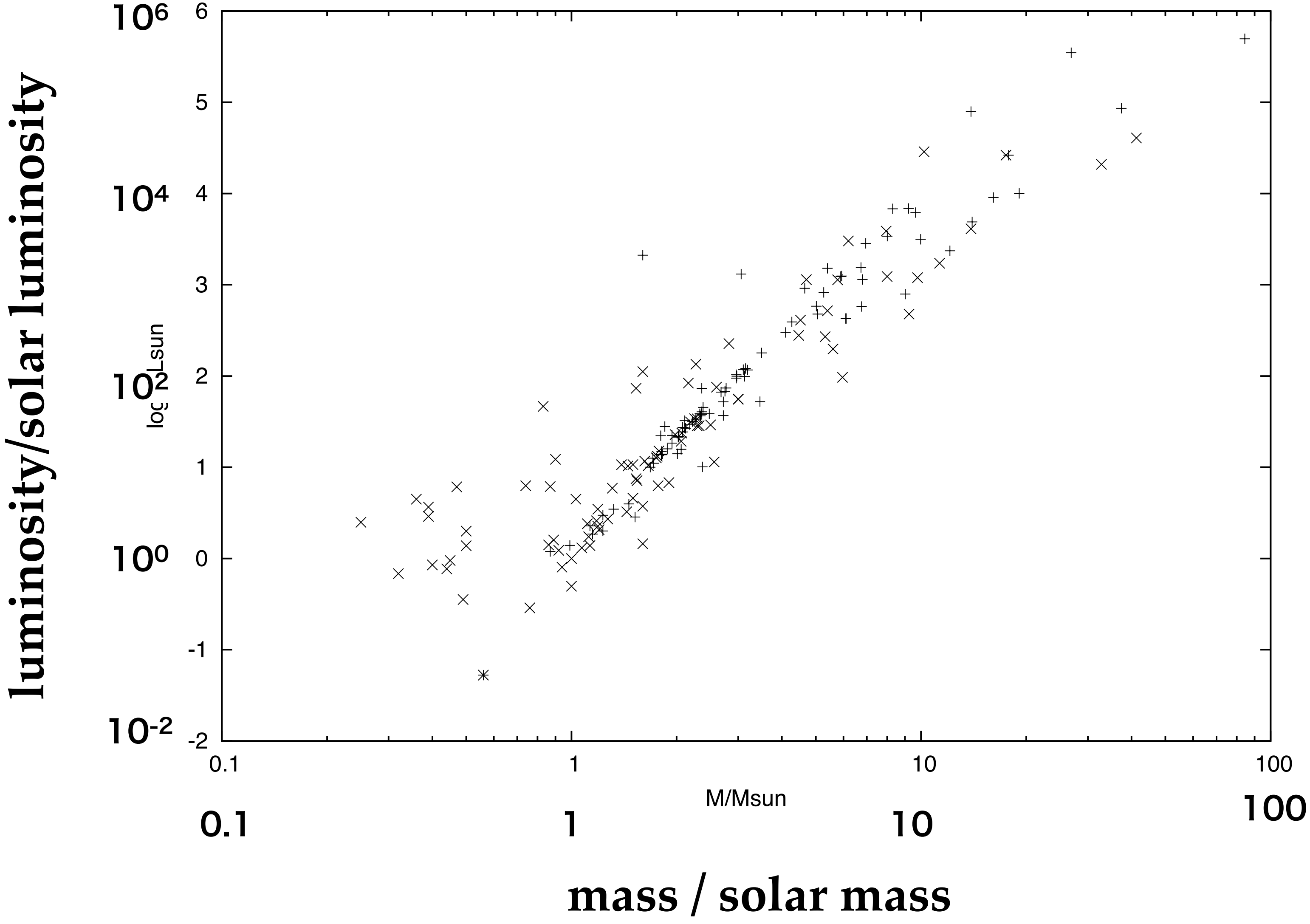
$$\mathrm{d}T/\mathrm{d}m = -\,3\kappa L_r/(64\pi^2 acT^3r^4)$$

$$\mathrm{LHS} \approx -T_c/M$$

$$\mathrm{RHS} \approx -3\langle\kappa\rangle\,(L/2)/(64\pi^2 ac)\,(T_c/2)^{-3}\,(R/2)^{-4}$$

$$\therefore\; L \approx \pi^2/(3\langle\kappa\rangle)\,\{acG^4/(k/m_u)^4\}\,\mu^{-4}\,M^3$$

Mass-Luminosity relation



Radiation from a Star

Stefan-Boltzmann law:

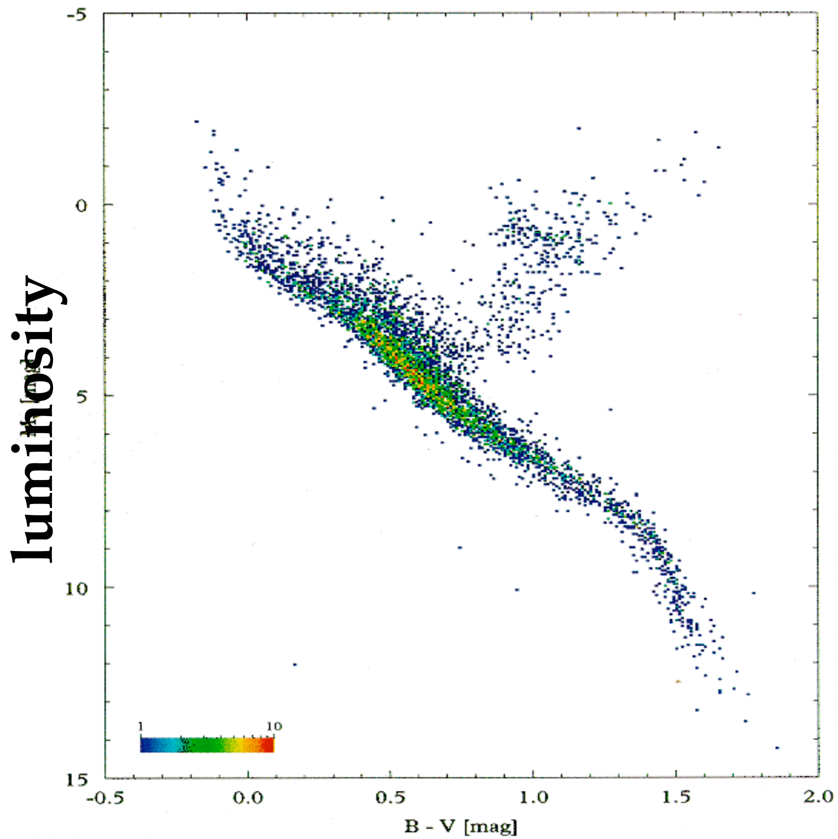
Radiation energy flux is proportional to T^4

$$L = A \int B_\lambda \mathrm{d}\lambda = A\ \sigma T_{\mathrm{eff}}{}^4$$

A = surface area (m^2) = $4\pi R^2$ （R: stellar radius）

Main Sequence

$$L \approx \pi^2/(3\langle\kappa\rangle)\ \{acG^4/(k/m_u)^4\}\mu^{-4}M^3$$



← surface temperature

Normalizing with the solar values,

$$\langle\kappa\rangle L/(\langle\kappa\rangle L)_{sun} \approx (M/M_{sun})^3$$

Since $\sigma T_{eff}^4 = L/(4\pi R^2)$,

$$(T_{eff}/T_{eff,sun})^4 = (L/L_{sun})(R/R_{sun})^{-2}$$

$$\approx (L/L_{sun})(M/M_{sun})^{-2}$$

$$\approx (L/L_{sun})^{1/3}$$

$$\therefore L/L_{sun} \propto (T_{eff}/T_{eff,sun})^{12}$$

Why are stars shining?

Nuclear fusion?

No!

Why are stars shining?

- Self gravity is supported by pressure.
- High gaseous pressure needs high temperature.
- Central temperature reaches 10^7 K.
- Energy flows from hot to cool regions.

Why are stars shining?

Simply because stars are hot !

Energy flows from hot to cool region.

Break

Stellar Evolution

Star: Energy loosing system

Energy loss = Cooling

Cooling timescale $= \int c_{\rm v} T \,\mathrm{d}m / L$

$\approx 10^7$ **yr for the Sun !**

Lifetime $\propto M/L \propto M^{-2}$

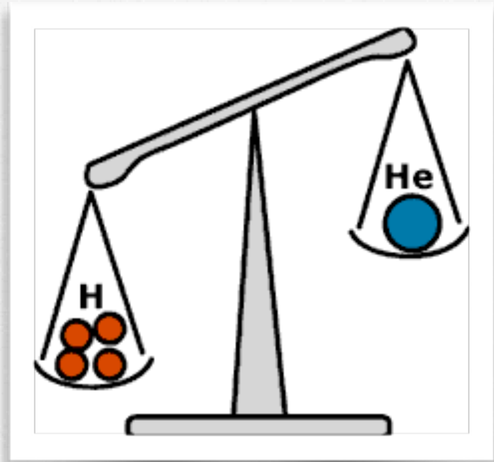
Necessity for sustaining mechanism

Nuclear fusion?

That's it!

Why can stars shine so long ?

mass = energy



H	atomic weight	1.008
He	atomic weight	4.002

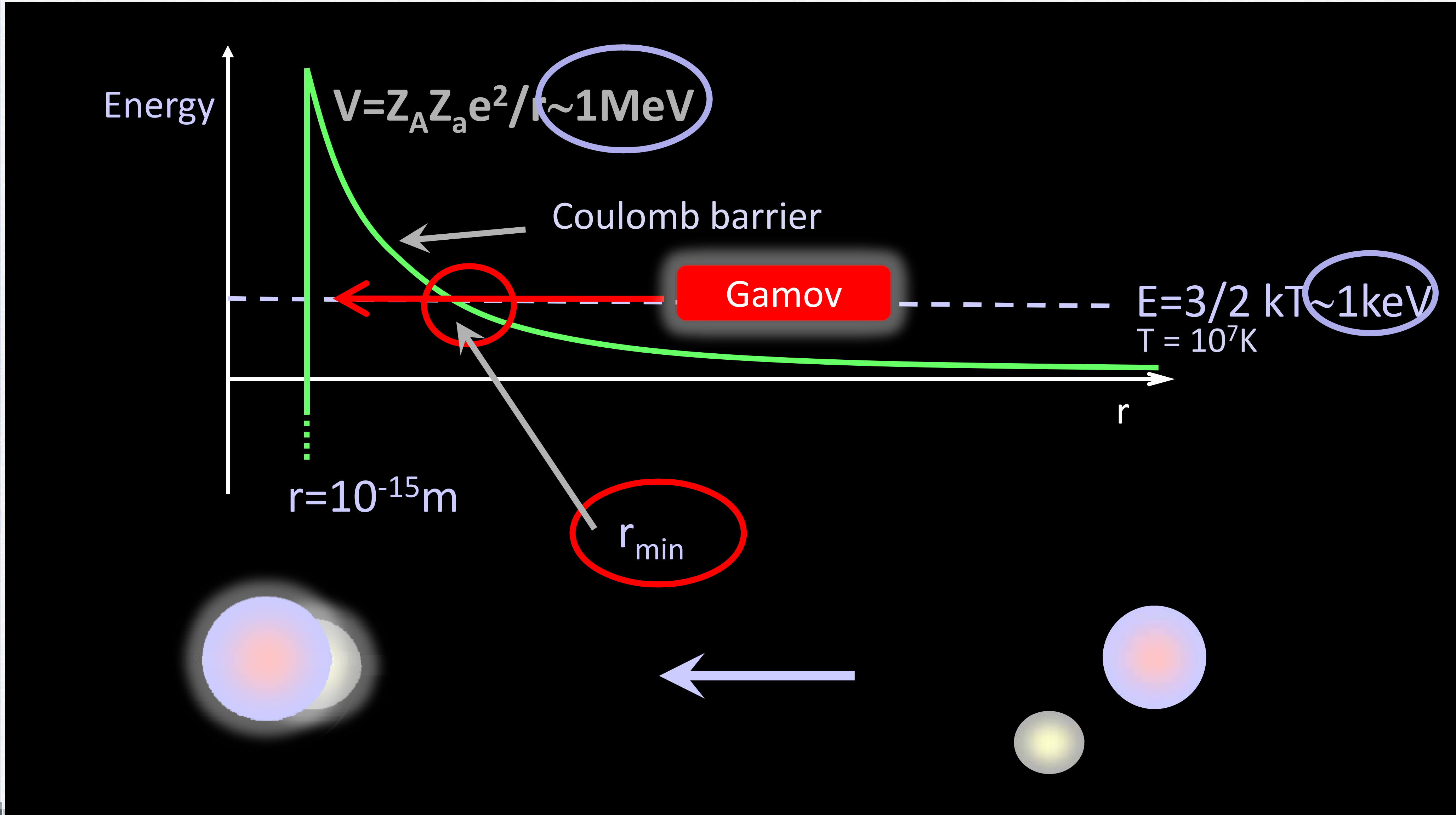
{4m(H) - m(He)}/4 = 0.007

Assume

1. **Solar composition: pure H**
2. **10% of H converted to He**

$$E_{\mathrm{nuc}} \simeq 0.007\ (0.1 M_{\mathrm{sun}}) c^2$$
$$\simeq 1.3\ 10^{44}\ \mathrm{J}$$
$$t_{\mathrm{nuc}} = E_{\mathrm{nuclear}} / L_{\mathrm{sun}}$$
$$\simeq 10^{10}\ \mathrm{yr}$$

Coulomb barrier to nuclear fusion



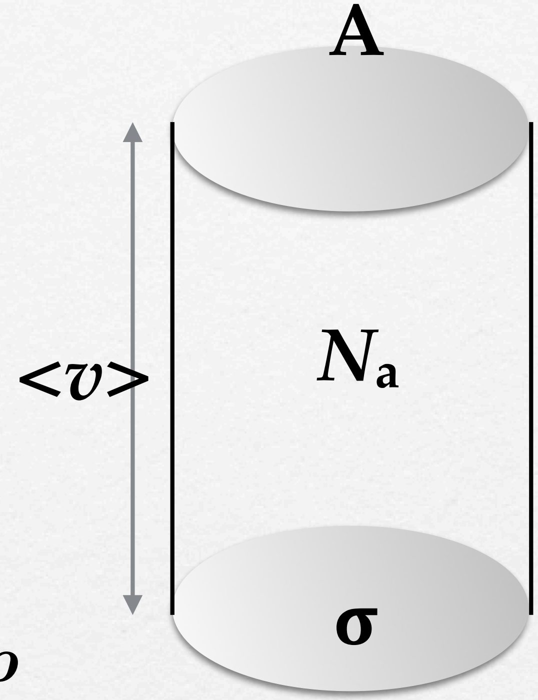
Nuclear reactions

$r = N_A N_a \langle \sigma v \rangle$

Maxwell-Boltzmann distribution for v

$$\langle \sigma v \rangle = \int_0^\infty \sigma(v) v \frac{1}{(2\pi m_\mu kT)^{3/2}} \mathrm{e}^{-E/kT} 4\pi p^2 \, dp$$

$$\langle \sigma v \rangle = \int_0^\infty \frac{8\pi}{m_\mu^{1/2}} \frac{1}{(2\pi kT)^{3/2}} \sigma(E) E \, \mathrm{e}^{-E/kT} \, dE$$



Cross section

Geometrical factor $\sigma = \pi\lambda^2 \approx \pi h^2/(2m_\mu E)$

Penetration factor $P = \exp(-2\pi^2 r_{\min}/\lambda)$

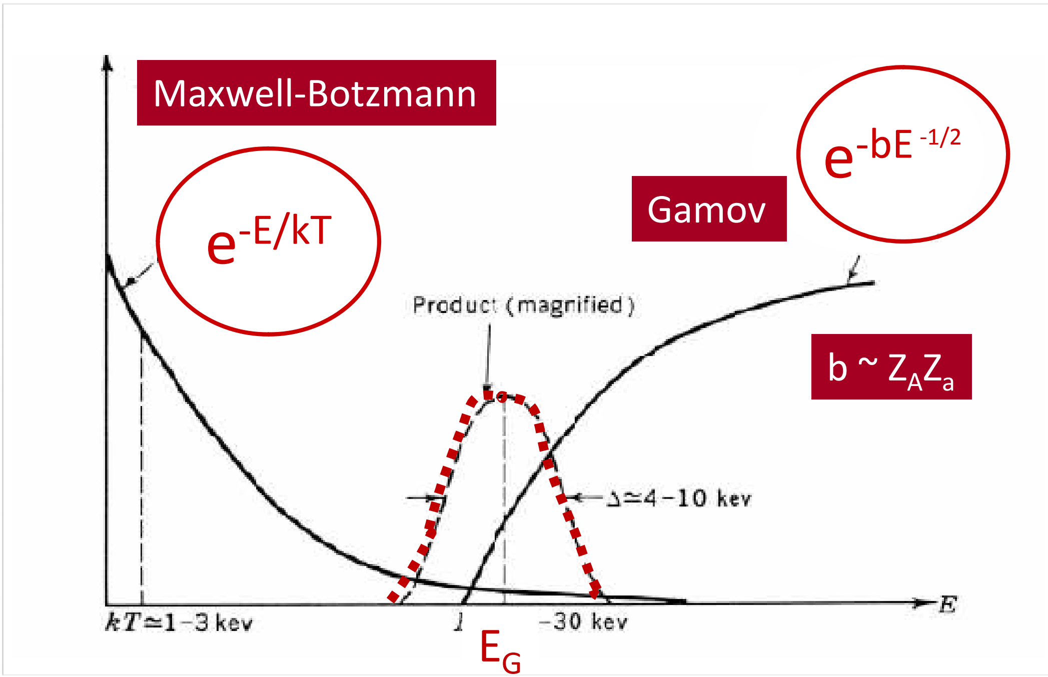
$\propto \exp(-\beta E^{1/2})$

Nuclear factor $S(E)$

$$\sigma(E) = E^{-1}\exp(-\beta E^{1/2})\, S(E)$$

Nuclear reactions

$$<\sigma v>= \left(\frac{8}{\pi m_\mu}\right)^{1/2} \left(\frac{1}{kT}\right)^{3/2} \int_0^\infty S(E) \exp\left(-\frac{E}{kT} - \beta E^{-1/2}\right) dE$$



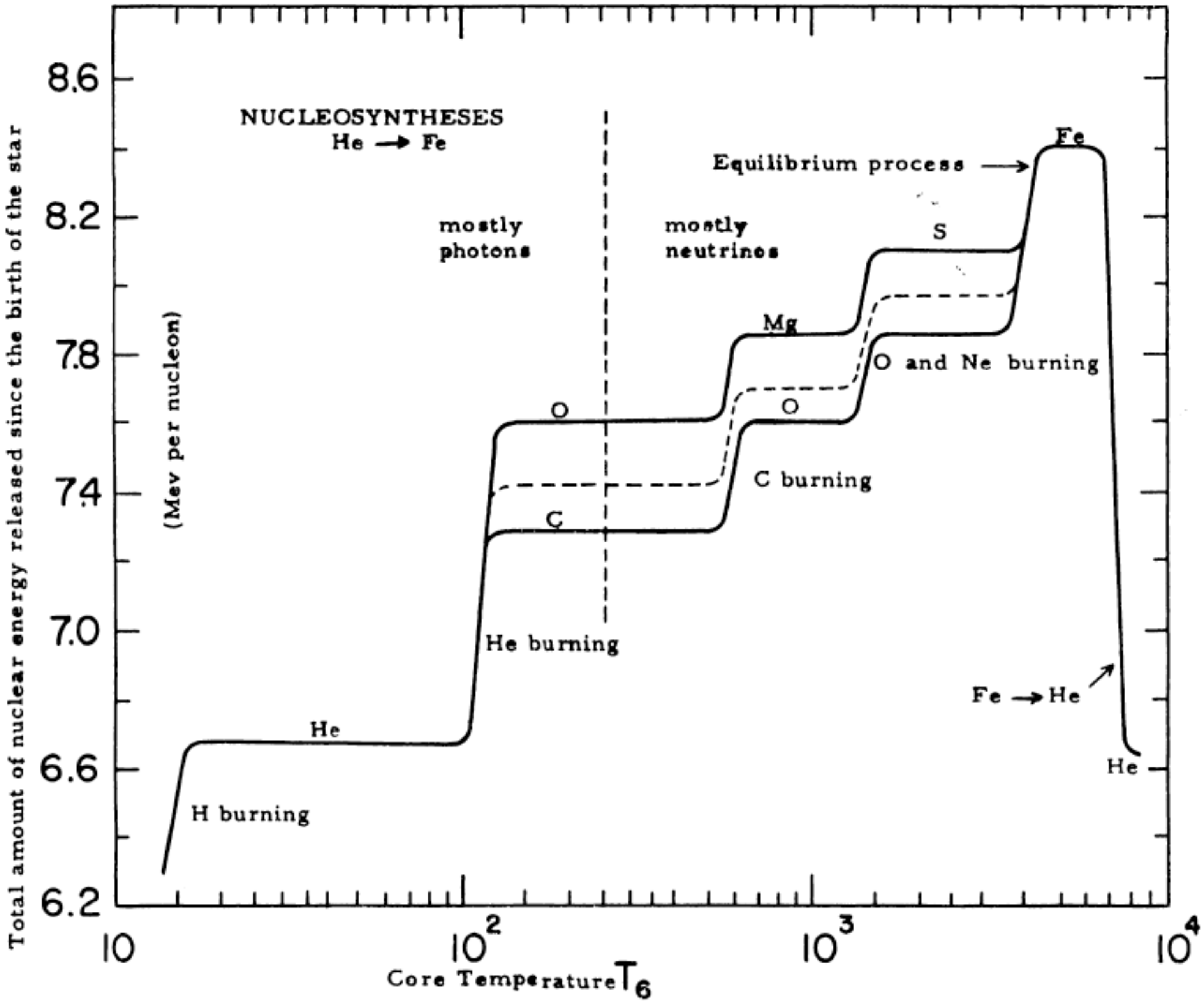
Gamow peak : $T \approx 10^7$ K

$\approx T_c$

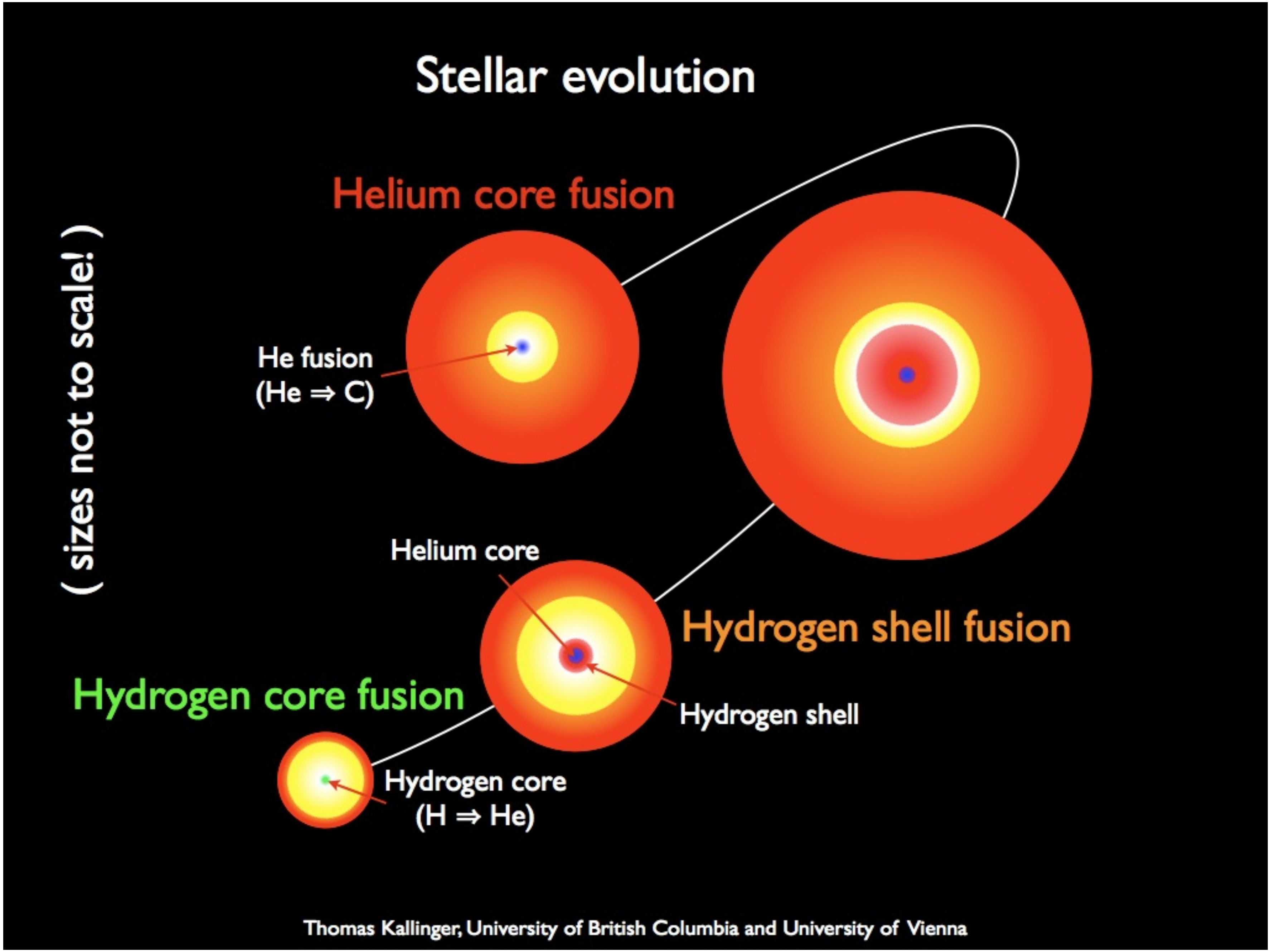
Is this a coincidence?

No! Stellar radius is adjusted so that

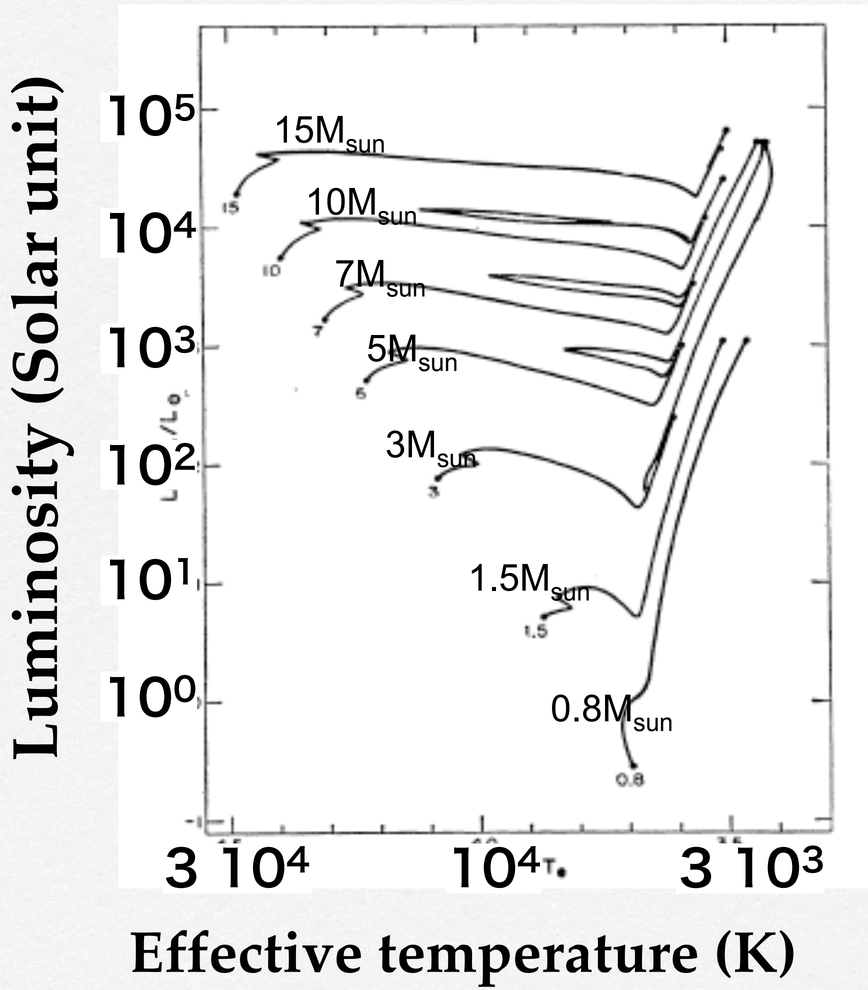
$$R \approx (k/\mu m_u)^{-1} GM/T_c$$



NUCLEOSYNTHESES
He → Fe
mostly photons
mostly neutrinos
Equilibrium process
Fe
S
Mg
O and Ne burning
O
C burning
O
C
He burning
He
H burning
Fe → He
He
8.6
8.2
7.8
7.4
7.0
6.6
6.2
Total amount of nuclear energy released since the birth of the star
(Mev per nucleon)
10
10^2
10^3
10^4
Core Temperature T6


Stellar evolution
Helium core fusion
He fusion
(He ⇒ C)
(sizes not to scale!)
Helium core
Hydrogen shell fusion
Hydrogen core fusion
Hydrogen shell
Hydrogen core
(H ⇒ He)
Thomas Kallinger, University of British Columbia and University of Vienna

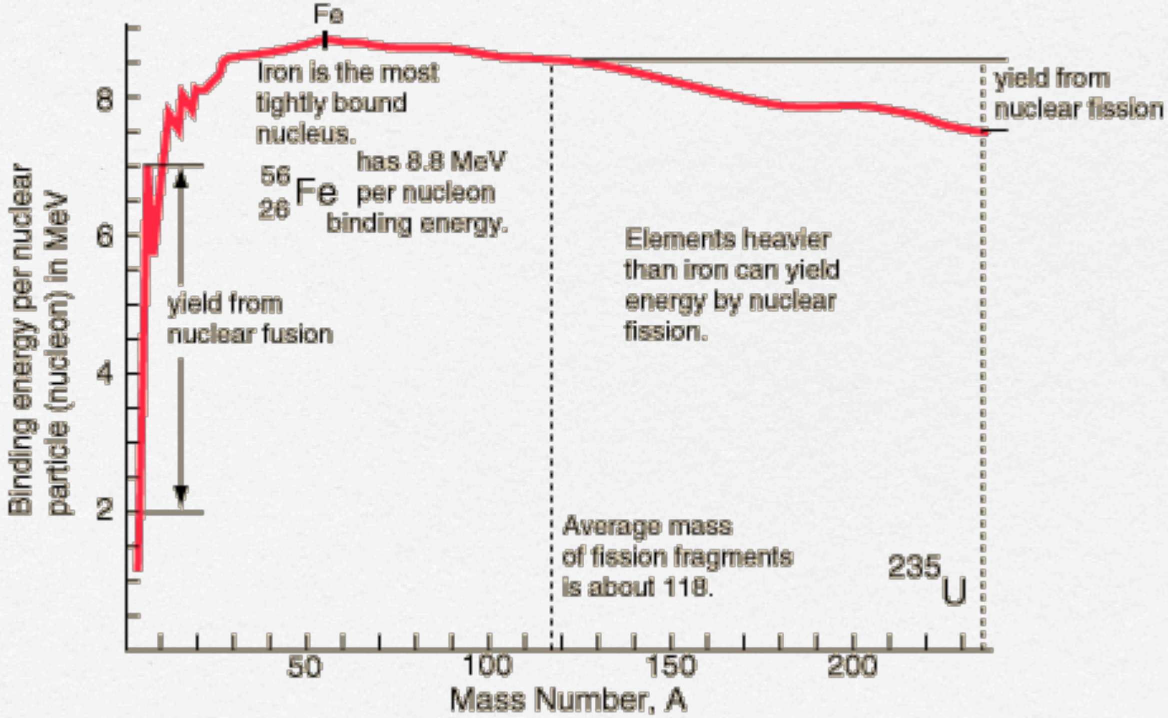
Stellar Evolution



- Mass-Luminosity relation
  $L \propto M^{\alpha}$
- $\tau \propto M/L \propto M^{1-\alpha}$
- Masin Sequence -> Red Giants

Why Main Sequence?

Energy liberated by He and heavier nuclei is ~1/10 of the case of H burning



Hydrogen

Four hydrogen nuclei（4^1H）are converted to a helium nucleus (^4He)

atomic weight of H = 1.008, so 4.032 by 4^1H
atomic weight of He = 4.002

Hence, liberated energy per nucleus is proportional to
(4.032 - 4.002)/4

Helium

Three helium nuclei（3^4He）are converted to a nucleus of carbon（^{12}C）

Atomic weight of He = 4.002, so 12.006 by 3^4He
Atomic weight of C = 12.000

Hence, liberated energy per nucleus is proportional to (12.006 - 12.000)/12

Lifetime of He burning will be shorter than that of H burning by a factor of
[(12.006 - 12.000)/12] / [(4.032 - 4.002)/4]

Essence of stellar evolution

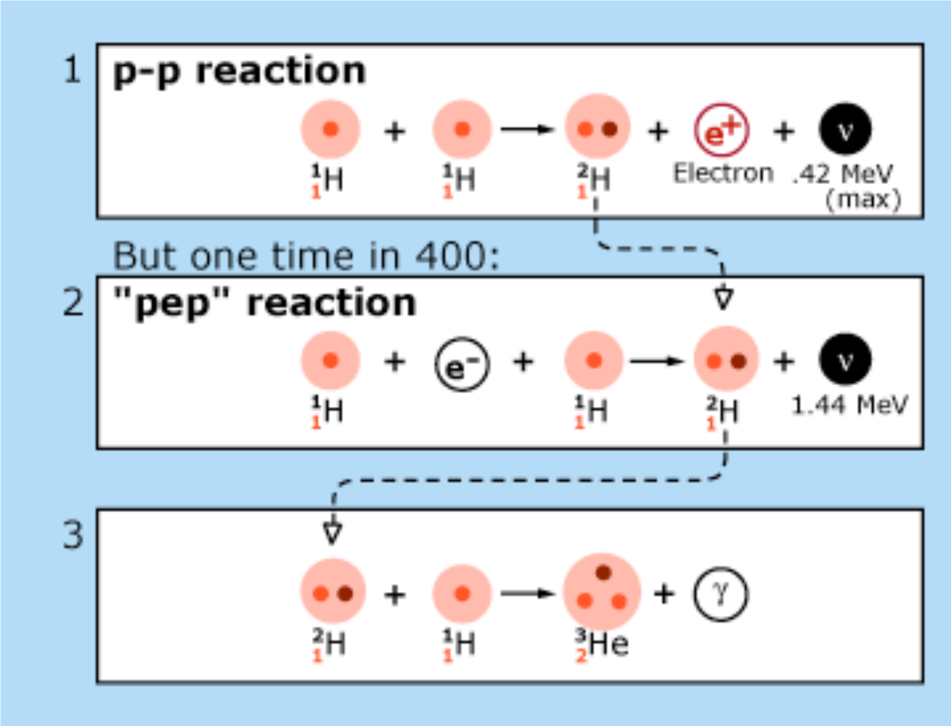
- Toward gravitational contraction
  - However, its timescale is not GM^2/RL
- Residence by nuclear reactions
  - Timescales are governed by nuclear reactions

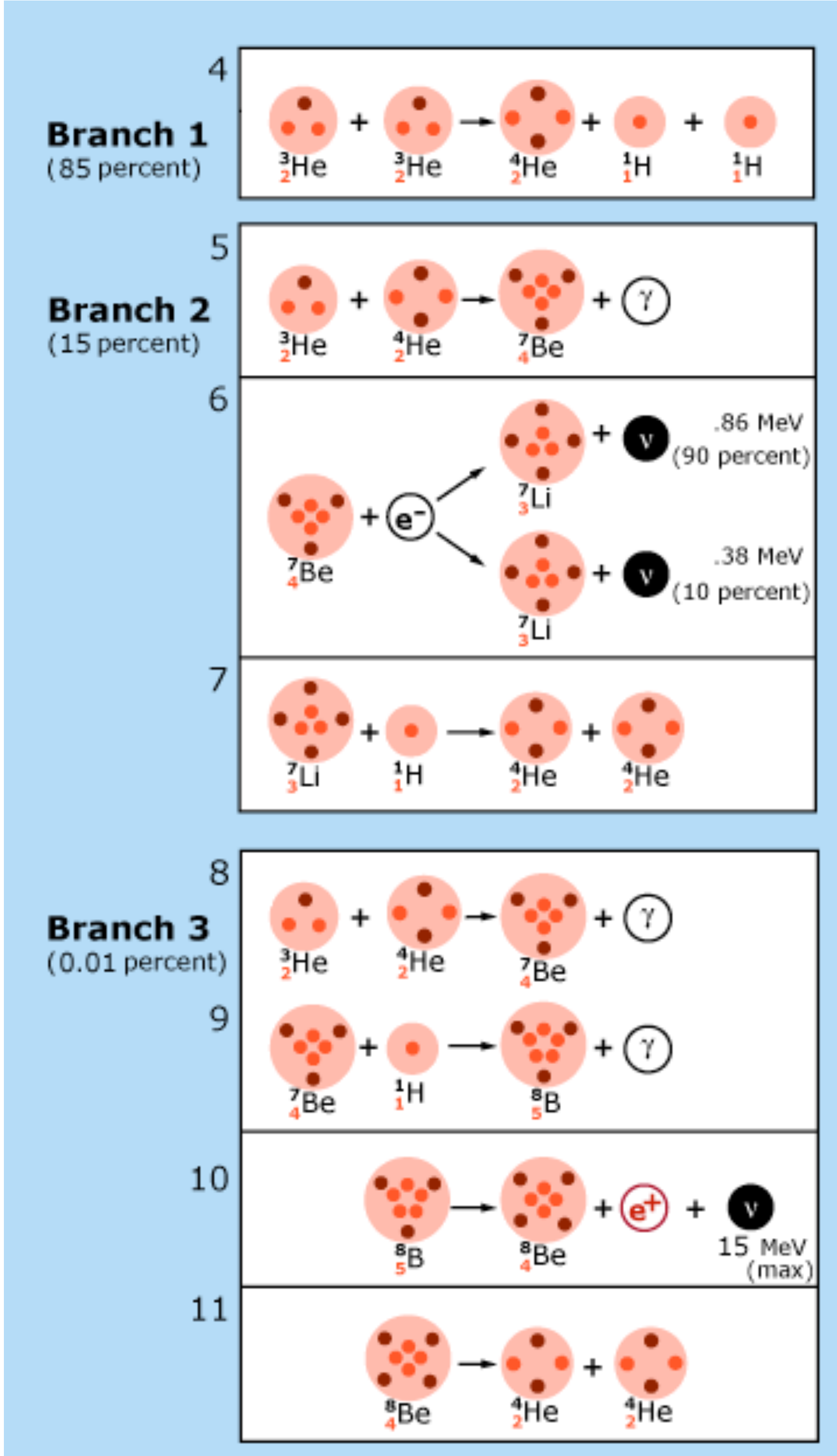
Break

Is the Sun burning indeed?

In proof of nuclear reaction

- photon：diffusion process; 10 million years required to be transported from the center to the solar surface
- neutrino：no interaction with matter; only 2 sec to reach the solar surface
- neutrino flux measurement is the only way to prove the nuclear reactions in the





1 MeV = 1.6 10^{-13} J

pp-chain

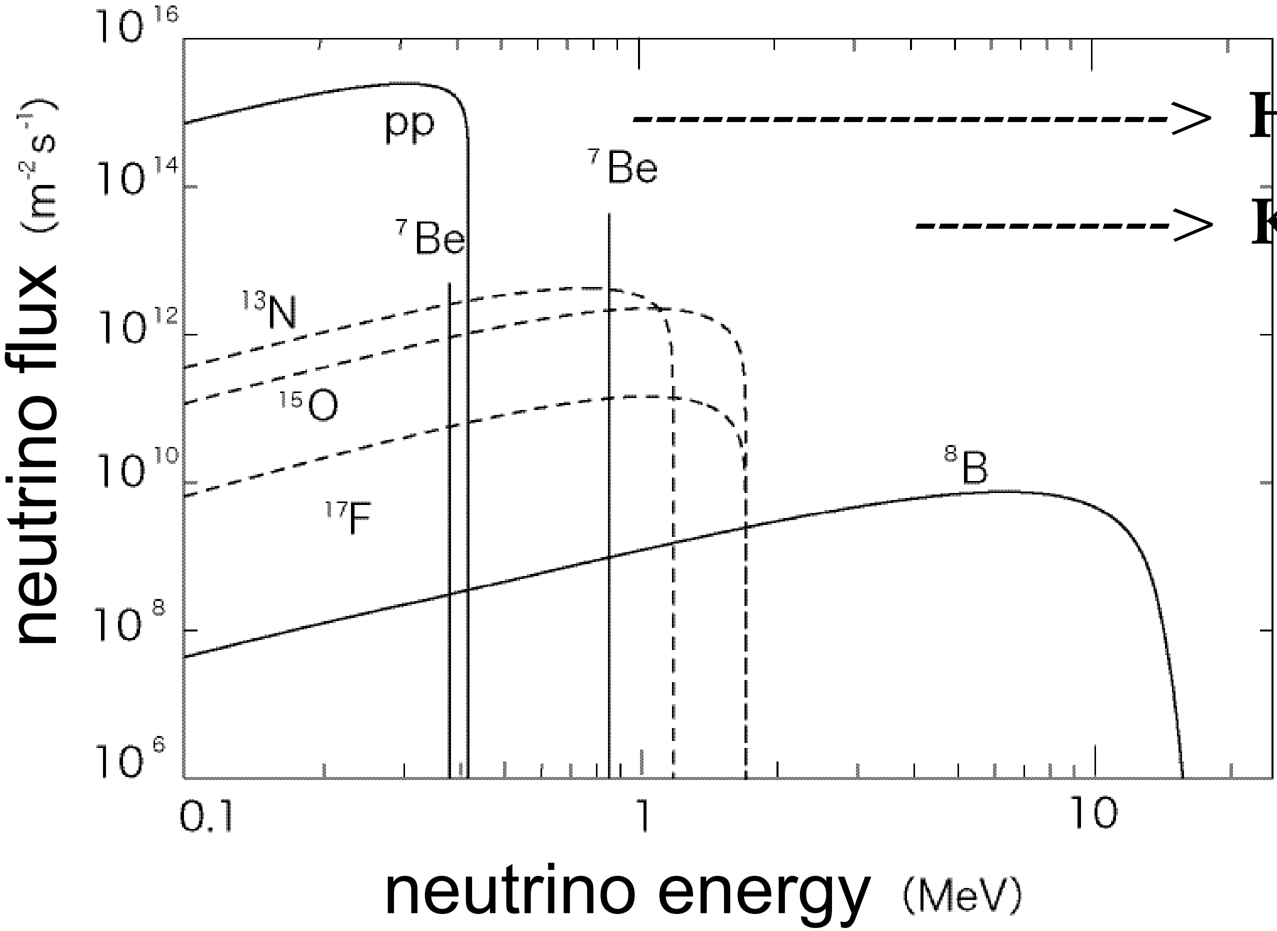
adopted from :

J.N. Bahcall,

Neutrinos from the Sun, Scientific American,

Volume 221, Number 1, July 1969, pp. 28-37.

Energy spectrum of solar neutrinos

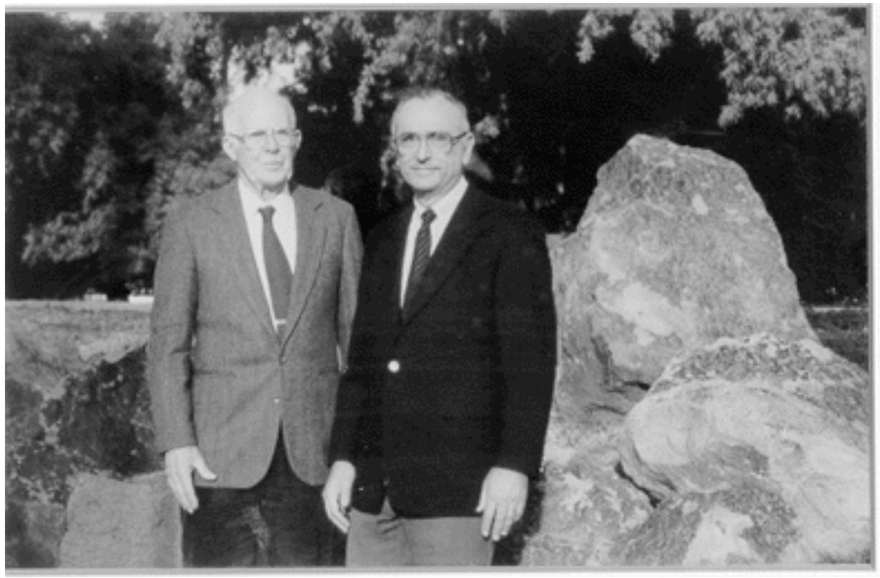


-------------------------> Homestake

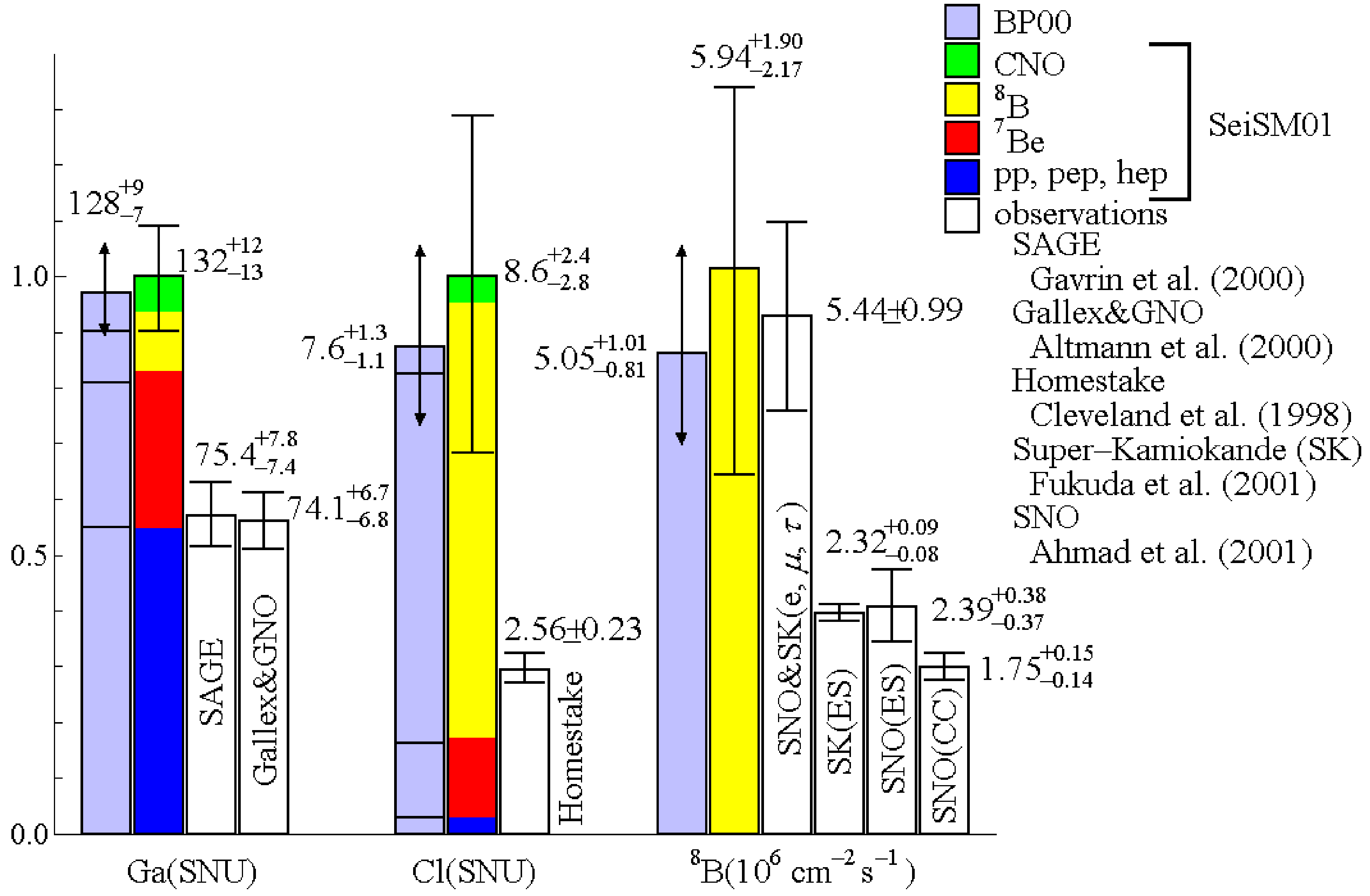
-----------> Kamiokande

Solar neutrino problem

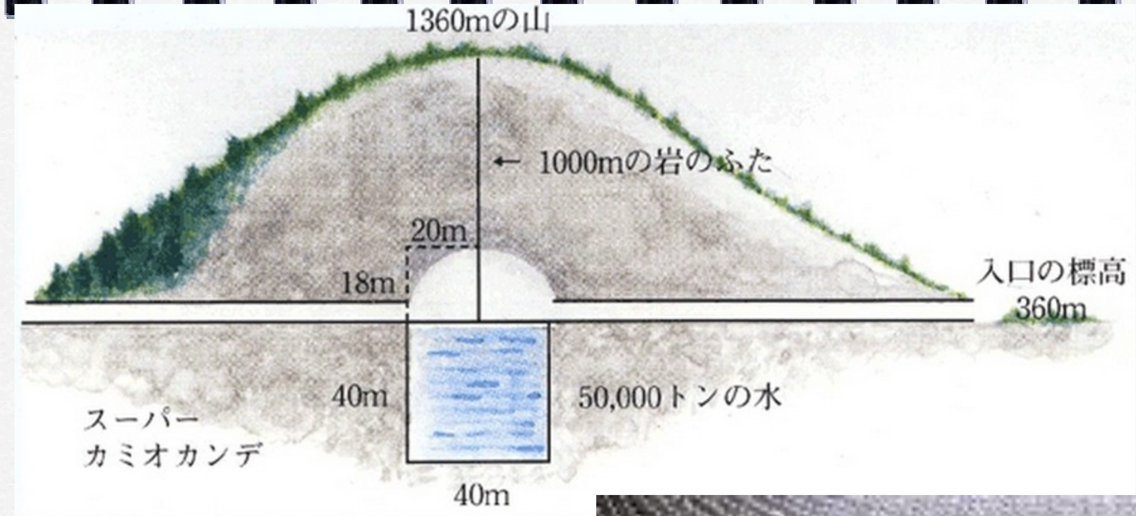
- pioneering work by R. Davis (1960-)
- Kamiokande (1987-)
- detected flux is about a half of theoretical prediction !
- experiments wrong？
- solar models wrong？
- particle physics wrong？



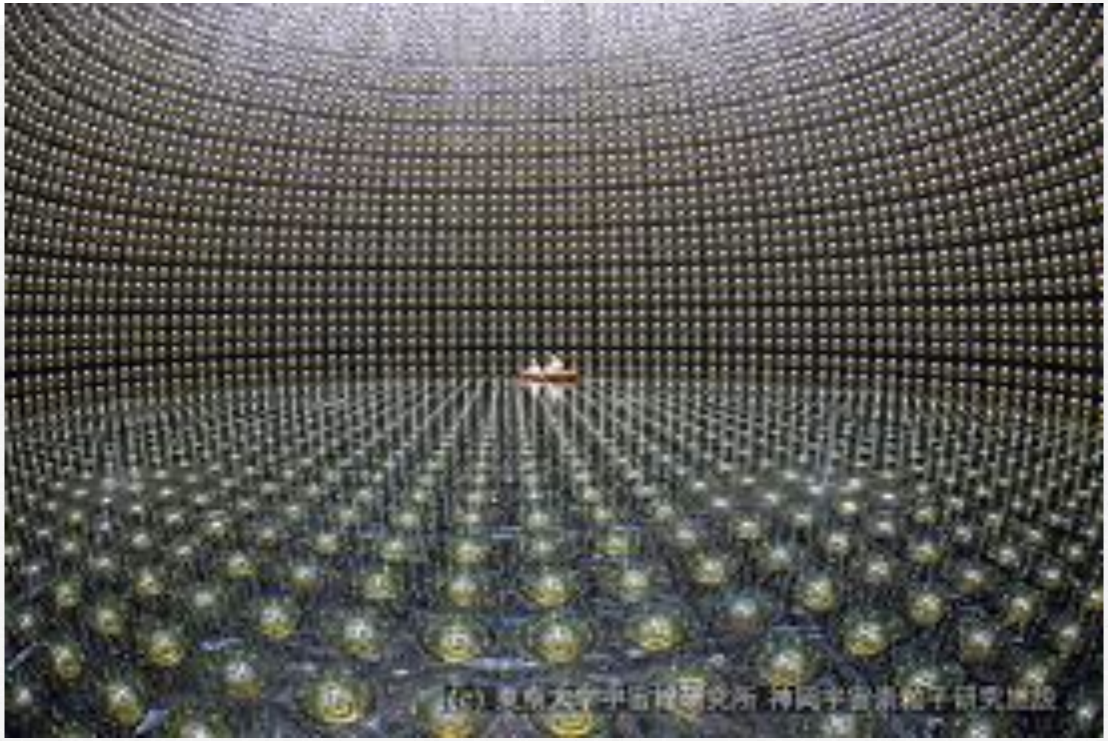
R. Davis & J. Bahcall

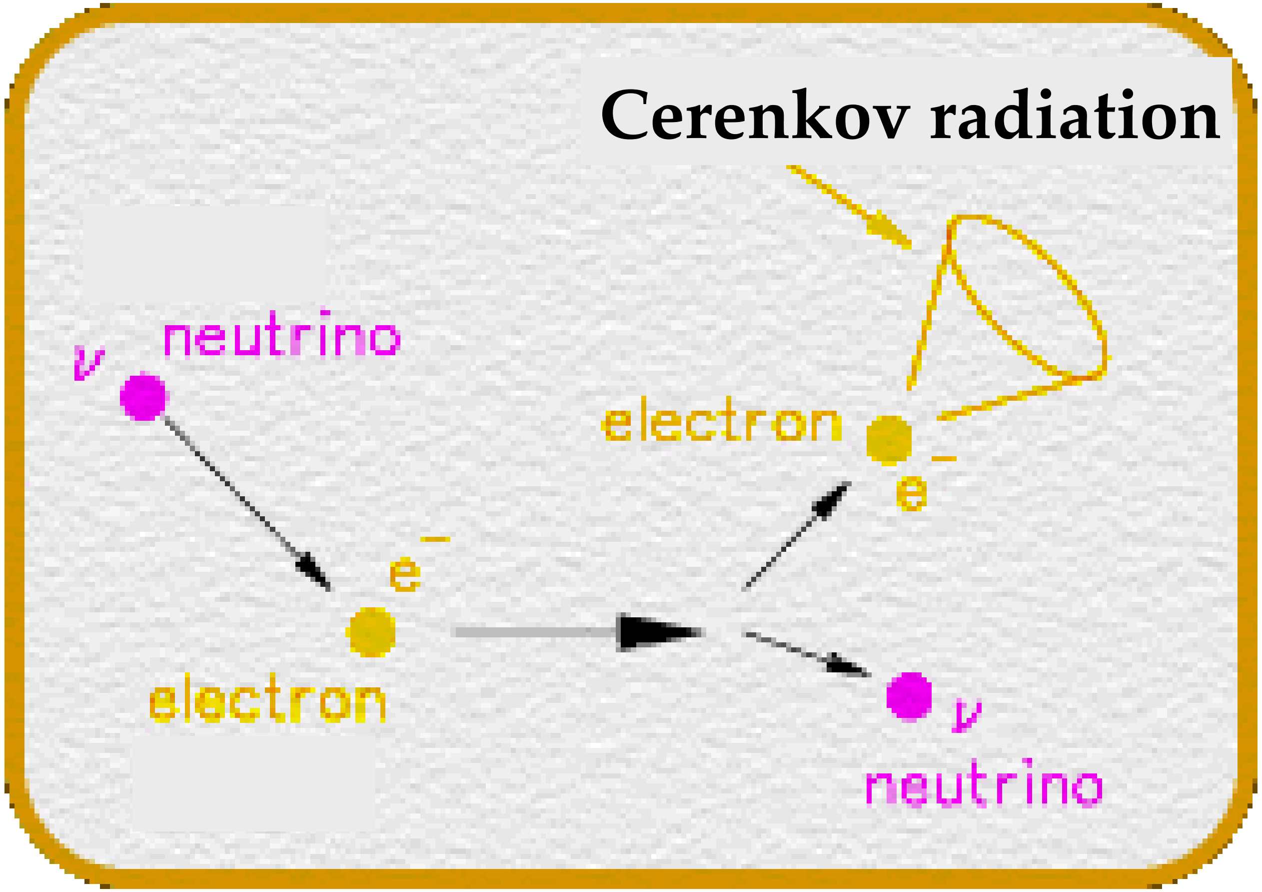


BP00
CNO
8B
7Be
pp, pep, hep
observations
SeiSM01
SAGE
Gavrin et al. (2000)
Gallex&GNO
Altmann et al. (2000)
Homestake
Cleveland et al. (1998)
Super–Kamiokande (SK)
Fukuda et al. (2001)
SNO
Ahmad et al. (2001)
1.0
0.5
0.0
128+9−7
132+12−13
75.4+7.8−7.4
74.1+6.7−6.8
SAGE
Gallex&GNO
7.6+1.3−1.1
8.6+2.4−2.8
2.56±0.23
Homestake
5.94+1.90−2.17
5.05+1.01−0.81
5.44±0.99
SNO&SK(e, μ, τ)
2.32+0.09−0.08
2.39+0.38−0.37
1.75+0.15−0.14
SK(ES)
SNO(ES)
SNO(CC)
Ga(SNU)
Cl(SNU)
8B(10^6 cm^−2 s^−1)

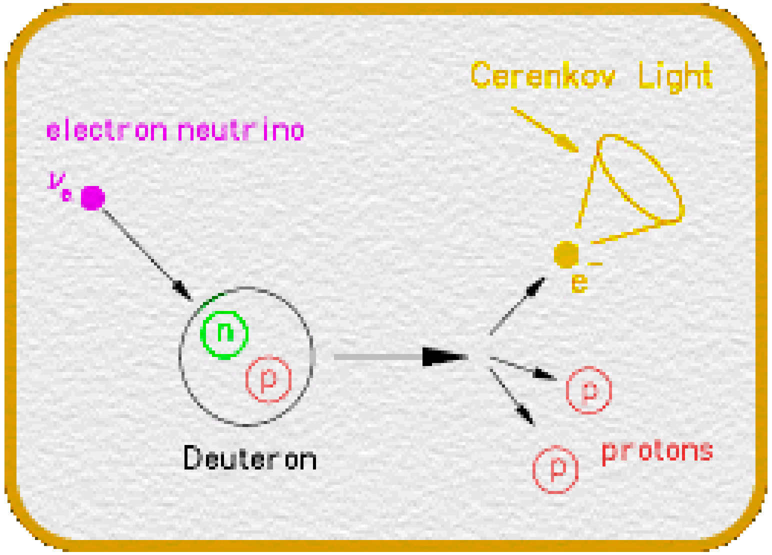


SuperKamiokande

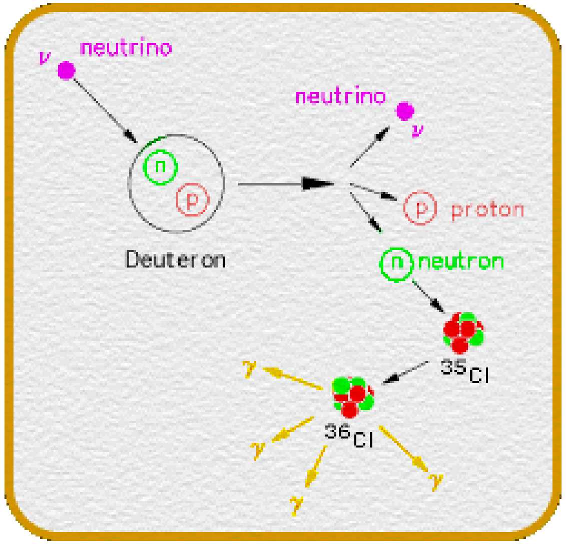




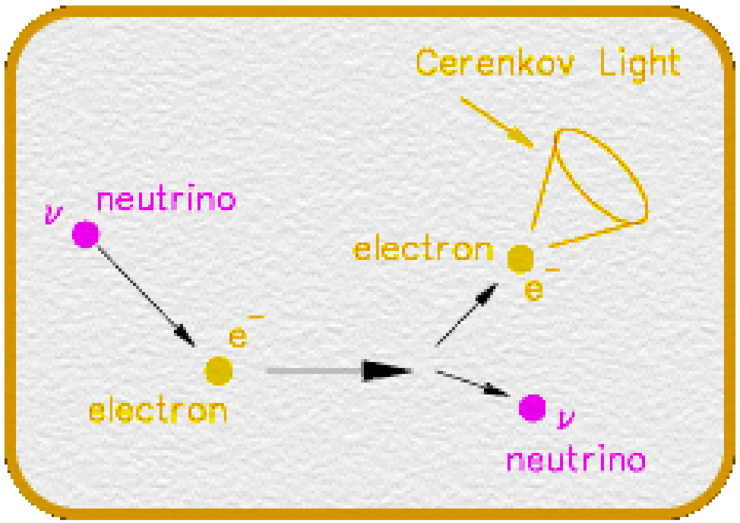
reaction occurring in SuperKamiokande



reaction I (CC mode)



reaction II (NC mode)



reaction III (ES mode)

Sudbury Neutrino Obs. experiment using D_2O

different sensitivity to neutrino types

SK (H_2O experiment)

- mainly sensitive to ν_e
- but slightly sensitive also to ν_μ &ν_τ

SNO (D_2O experiment)

- ES mode：the same as SK experiment
- CC mode：only sensitive to ν_e
- NC mode：sensitive to all of ν_e, ν_μ & ν_τ

$$F_{SK} = F_{\nu e} + 0.15\,(F_{\nu\mu} + F_{\nu\tau})$$

$$F^{CC}_{SNO} = F_{\nu e}$$

$$F^{NC}_{SNO} = F_{\nu e} + (F_{\nu\mu} + F_{\nu\tau})$$

LHS : observables

unknowns : $F_{\nu e}$ & $(F_{\nu\mu} + F_{\nu\tau})$

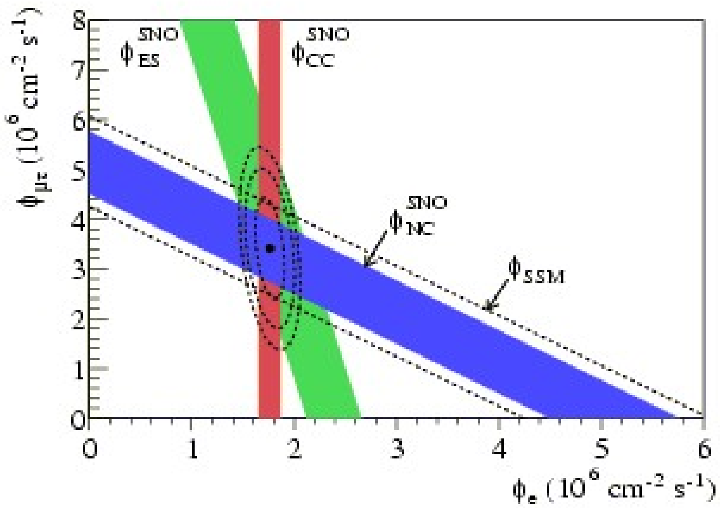


FIG. 3: Flux of ^{8}B solar neutrinos which are μ or τ flavor vs flux of electron neutrinos deduced from the three neutrino reactions in SNO. The diagonal bands show the total ^{8}B flux as predicted by the SSM [11] (dashed lines) and that measured with the NC reaction in SNO (solid band). The intercepts of these bands with the axes represent the $\pm 1\sigma$ errors. The bands intersect at the fit values for ϕ_e and $\phi_{\mu\tau}$, indicating that the combined flux results are consistent with neutrino flavor transformation assuming no distortion in the ^{8}B neutrino energy spectrum.

solution to the solar neutrino problem:

neutrino oscillations

<-- non-zero mass of neutrinos!

Solar neutrino problem

- Energy source = nuclear reactions
- 4H -> He + neutrinos
- photon: diffusion
- time scale = several millions years
- neutrino: no interaction

End of Lecture I