天体物理学特論I (2014) Report (Ver.2), due 9/12 (Fri)

Submit the report to me or the office of Dept. of Astronomy

I. Consider a star sustained by the pressure of radiation, P_{rad}, and ideal gas, P_{gas}, with mean molecular weight μ.

i) Write down the total pressure $P = P_{gas} + P_{rad}$ for this star using the Boltzmann constant k, atomic mass unit m_u, and radiation constant a (= $4\sigma/c$), where σ is the Stephan-Boltzmann constant and c is the velocity of light.

ii) Define β (beta) by the ratio of the gas pressure to the total pressure as $\beta = P_{gas} / P$. Then using the relation $P = P_{gas} / \beta = P_{rad} / (1-\beta)$ show that

$$T = \left[\frac{3k(1-\beta)}{m_u a \mu \beta}\right]^{1/3} \rho^{1/3}.$$

iii) Then show that

if we write P in the form $P = K\rho^{4/3}$ ... (A),

$$K = \left[\frac{3k^4(1-\beta)}{m_u^4 a \mu^4 \beta^4}\right]^{1/3} \quad \dots (B).$$

In the followings, we assume β is constant throughout the star. Then K is constant and the equation (A) becomes a polytropic relation, and the star can be represented by the polytrope with the polytropic index n =3.

iv) For the n=3 polytrope, it is known that stellar mass M is related to K by the equation $M = 8.08\,\pi\,(K/\pi G)^{3/2}$, where G is the gravitational constant. Derive the following equation and determine the constant C: $1-\beta = C\,(M/M_{\odot})^2\,\mu^4\,\beta^4$.

v) Determine the values of β for M= 5, 10, 20, 50, 100, 1000, and 10^4 $M_{\odot}$, when μ = 0.59.

II. If β is nearly zero and contribution of gas is neglected, one can write the total energy of the star E as follows (ρ_c is the central density):

$E = E_{int} + E_{grav}$, where $E_{int} = k_1 K M \rho_c^{1/3}$, $E_{grav} = - k_2 G M^{5/3} \rho_c^{1/3}$ are internal and gravitational energy, respectively, and $k_1 = 1.75579$ and $k_2 = 0.639001$.

Here photon entropy per baryon s_r is uniform inside star, and then K is represented in terms of photon entropy per baryon

$$s_r = \frac{4 m_u a T^3}{3\rho} \quad \text{as} \quad K = \frac{a}{3}\left(\frac{3 s_r}{4 m_u a}\right)^{4/3} .$$

Note that this K can be obtained from eq.(A) by using the relation $\beta = P_{gas}/P \cong 4\, k / (\mu\, s_r) << 1$.

i) Equilibrium of this star is obtained by $\partial E / \partial \rho_c = 0$.

Show that in equilibrium, radiation entropy of the star is uniquely determined for a given M. Then calculate s_r / k for $M = 10^4, 10^5, 10^6\, M_\odot$, with 3 significant digits（有効数字３桁）.

ii) If $dM/d\rho_c \geq 0$, a star is stable. Discuss the stability of the equilibrium state calculated above.

Let's consider first order term of the contribution of gas, ΔE_{int} , in the $E = E_{int} + \Delta E_{int} + E_{grav}$. This approximately gives the following equation:

$$E = k_1^{'} K M \rho_c^{1/3} + \frac{4 k_1 K M}{3}\left(\frac{k}{s}\right)\rho_c^{1/3} \ln \rho_c - k_2 G M^{5/3} \rho_c^{1/3} \;...\; (C).$$

where $k_1^{'}$ is a positive number not very much different from k_1 and $s = s_r + s_{gas} \cong s_r$.

iii) Discuss the stability of the equilibrium state for this E.

III. For a very massive star with $M > \sim 10^4 M_{\odot}$, it is known that the effects of general relativity (GR) is not completely negligible. The first order GR correction is given by adding the correction term

$$\Delta E_{GR} = -\frac{k_4 G^2}{c^2} M^{7/3} \rho_c^{\ 2/3}, \quad k_4 = 0.9183,$$

to the equation (C) in the last page.

Discuss the stability of this star, and find the conditions for the instability if exist.

IV. From the Journal papers mentioned in (or you think related to) this lecture, pick up one and

i) describes the contents of the paper with your words by a length of more than A4 half page in English or more than A4 one page in Japanese.

ii) discuss the implications and uncertainties in the results of the paper by a length of more than A4 one page in English or more than A4 two pages in Japanese.

Make sure to submit a copy of the journal paper with this report.

Have a nice summer !