

**Search for Invisible Companions  
(Planets, WD, NS, BH)  
based on Frequency Modulation**

# Binary

**Fundamental source of information of stellar physics**

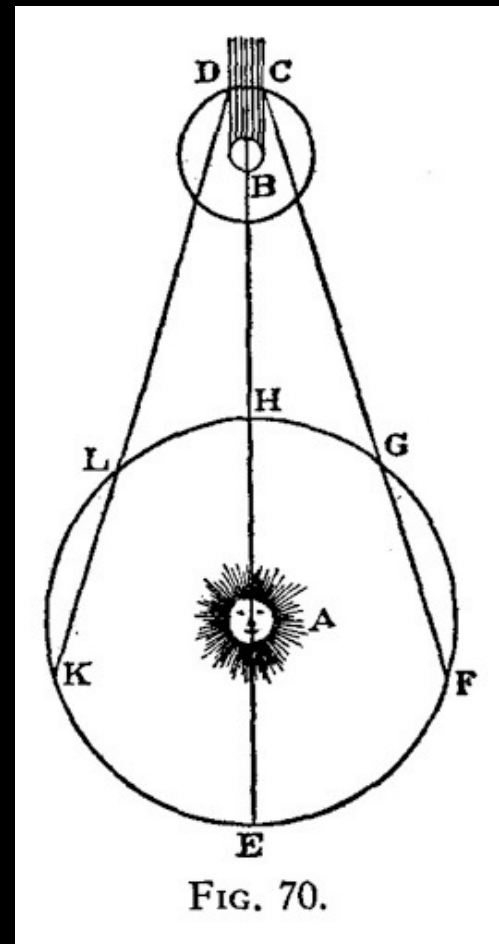
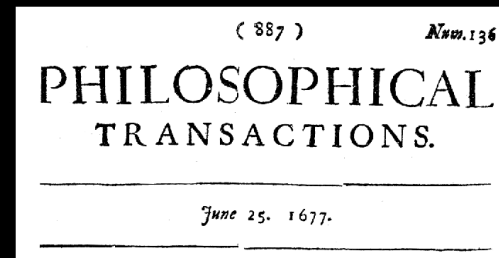
**Spectroscopic observations**

- telescope time allocation
- long time span to cover the orbital phase

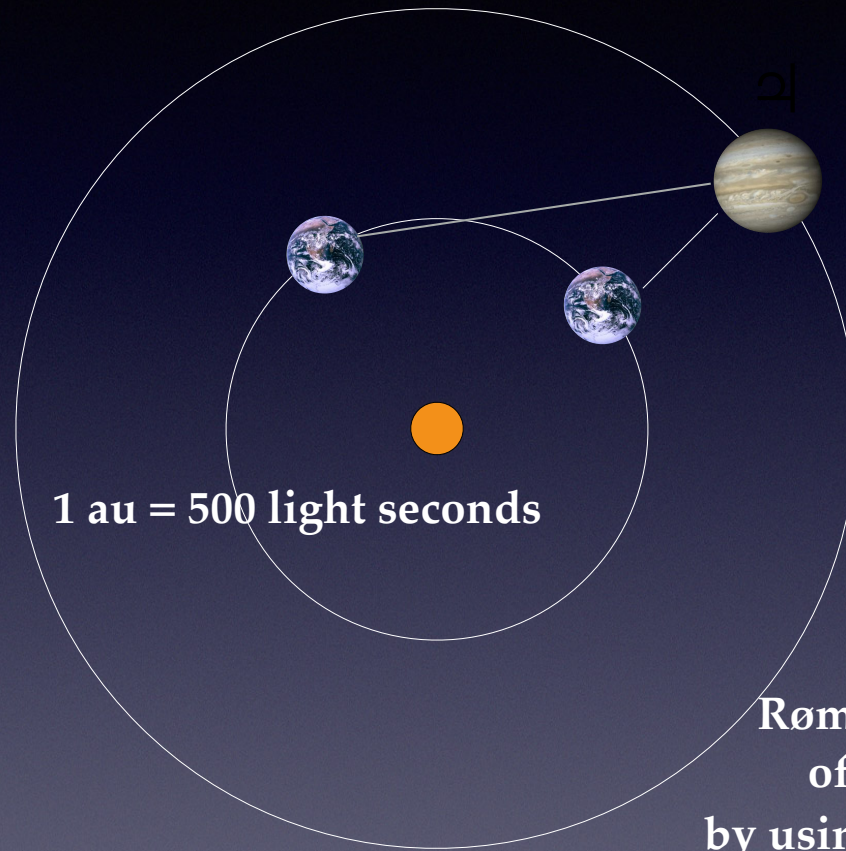
# Search for invisible exotic components

- Planets around early-type stars
- Brown dwarfs
- White dwarfs
- Neutron stars
- Stellar origin black holes

# Ole Roemer's measurement of the speed of light (1677)



# “light-time effect”

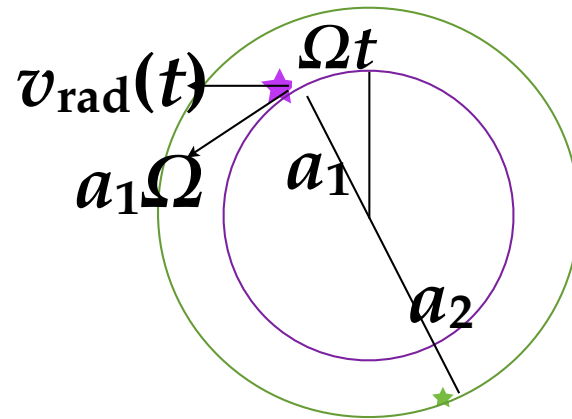


Rømer's measurement  
of the light speed  
by using Io's eclipse timing

# Clocks in a binary system

The simplest case: circular orbital motion

$$v_{\text{rad}}(t) = a_1 \Omega \sin i \cos \Omega t$$



$i$  : inclination angle

 observer

## Observed luminosity variation:

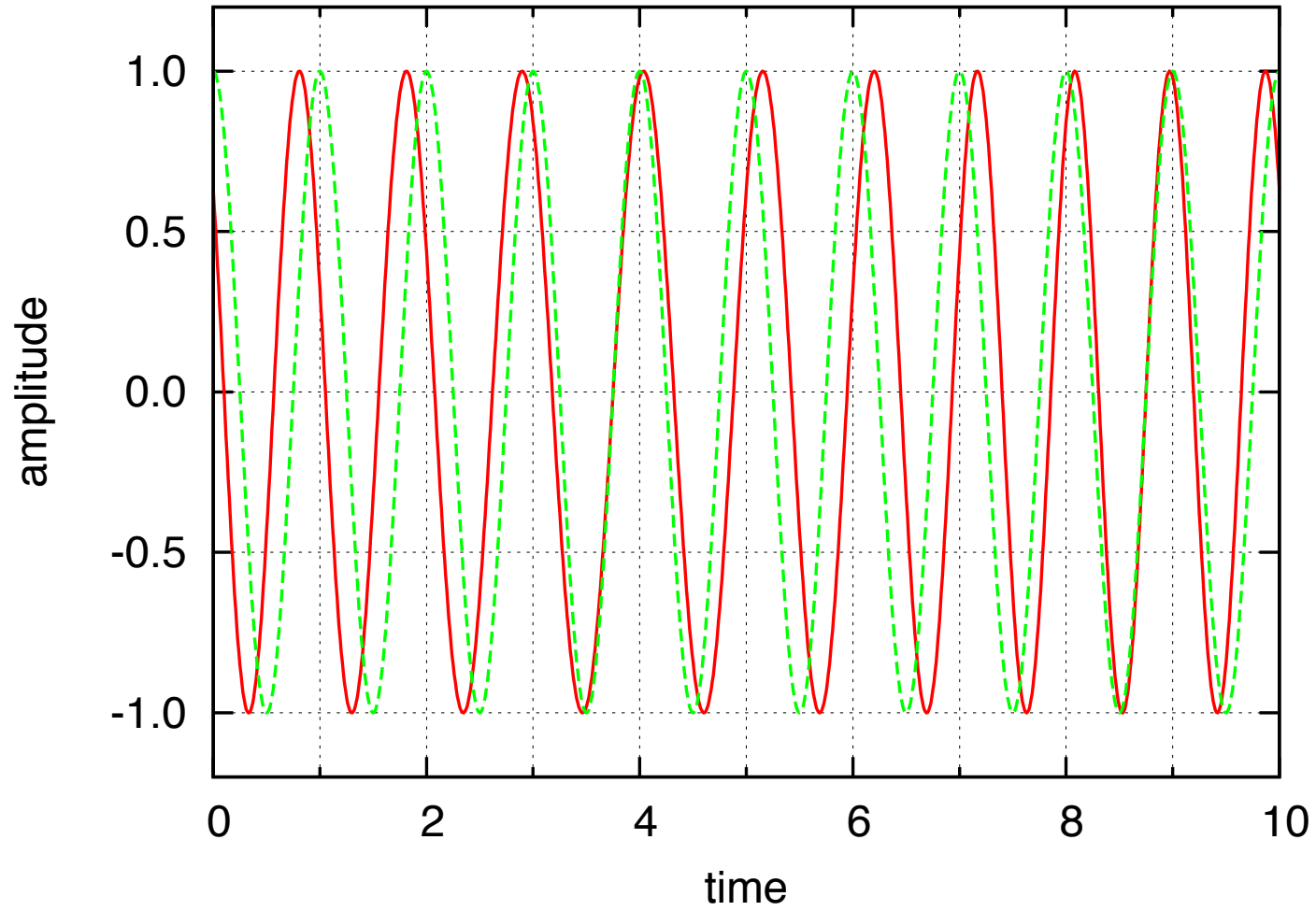
$$\Delta L(t) \propto \cos \left\{ \omega_0 \left[ t - c^{-1} \int v_{\text{rad}}(t') dt' \right] + \phi \right\}$$

arrival time delay

## Instantaneously observed frequency:

$$\omega_{\text{obs}}(t) = \omega_0 [1 - v_{\text{rad}}(t)/c]$$

# phase modulation





**Estimate of the amplitude of phase modulation:**

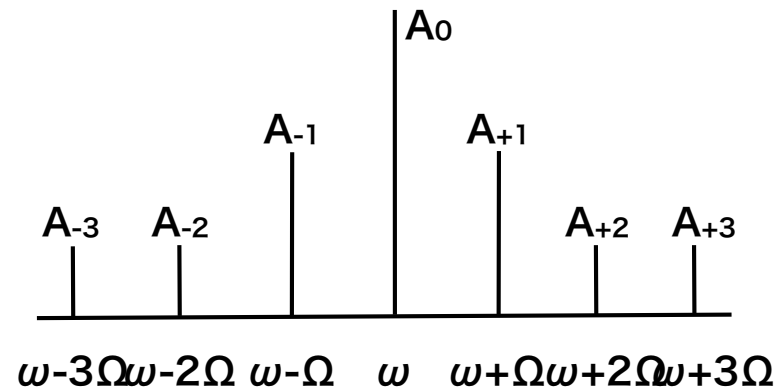
$$\begin{aligned} (v_{\text{rad}}/c)_{\text{max}} &= a_1 \Omega \sin i / c \\ &= (2\pi GM)^{1/3} c^{-1} q(1+q)^{-2/3} P_{\text{orb}}^{-1/3} \sin i \\ &\simeq 10^{-3} \end{aligned}$$

$$\begin{aligned} \omega_0 \left( \int v_{\text{rad}}/c \, dt \right)_{\text{max}} &= a_1 \omega_0 \sin i / c \\ &= (2\pi GM)^{1/3} c^{-1} q(1+q)^{-2/3} P_{\text{orb}}^{2/3} P_{\text{osc}}^{-1} \sin i \\ &\equiv \alpha \end{aligned}$$

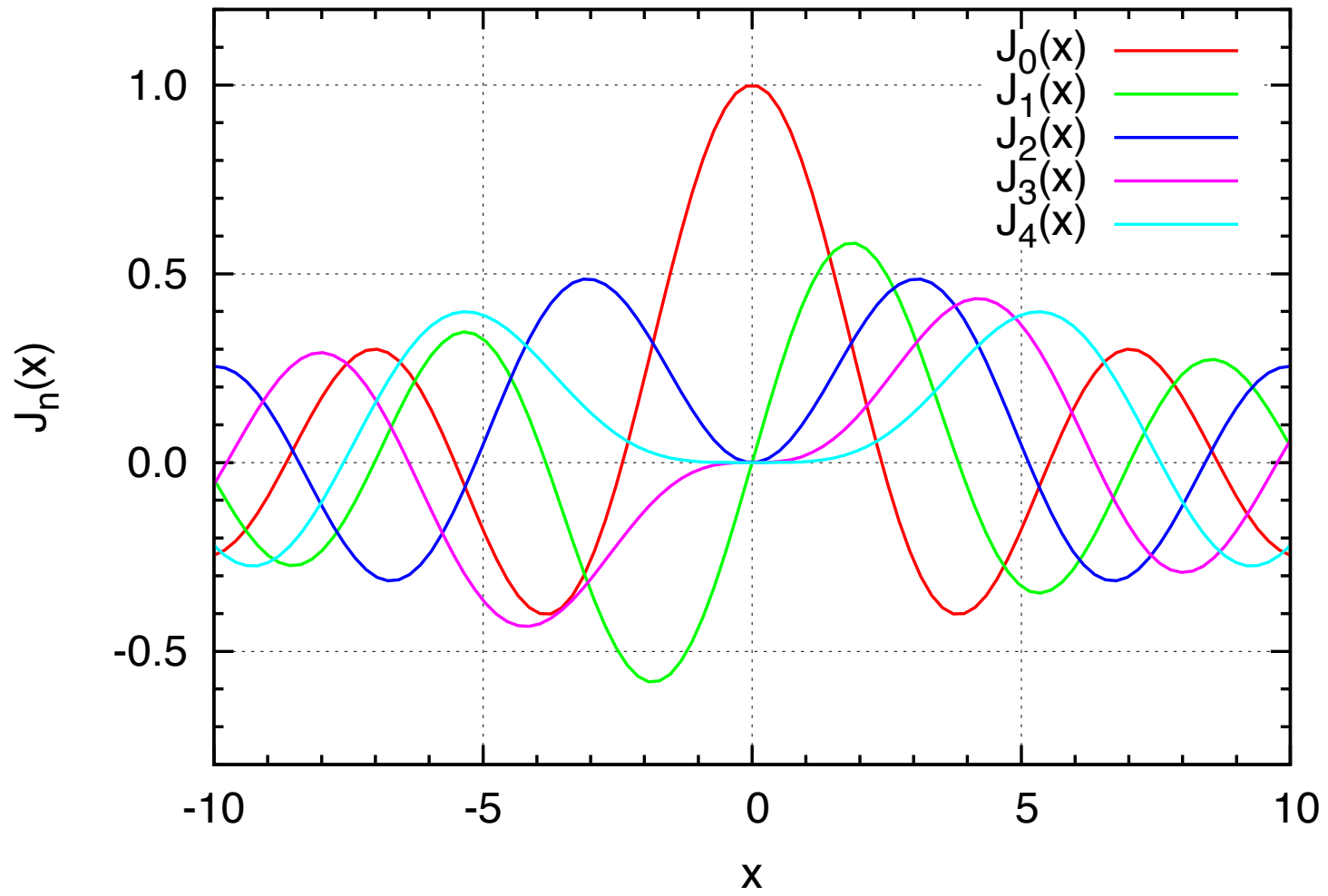
## Fourier transform of phase modulation

$$\cos(\omega_0 t + \phi - \alpha \sin \Omega t)$$

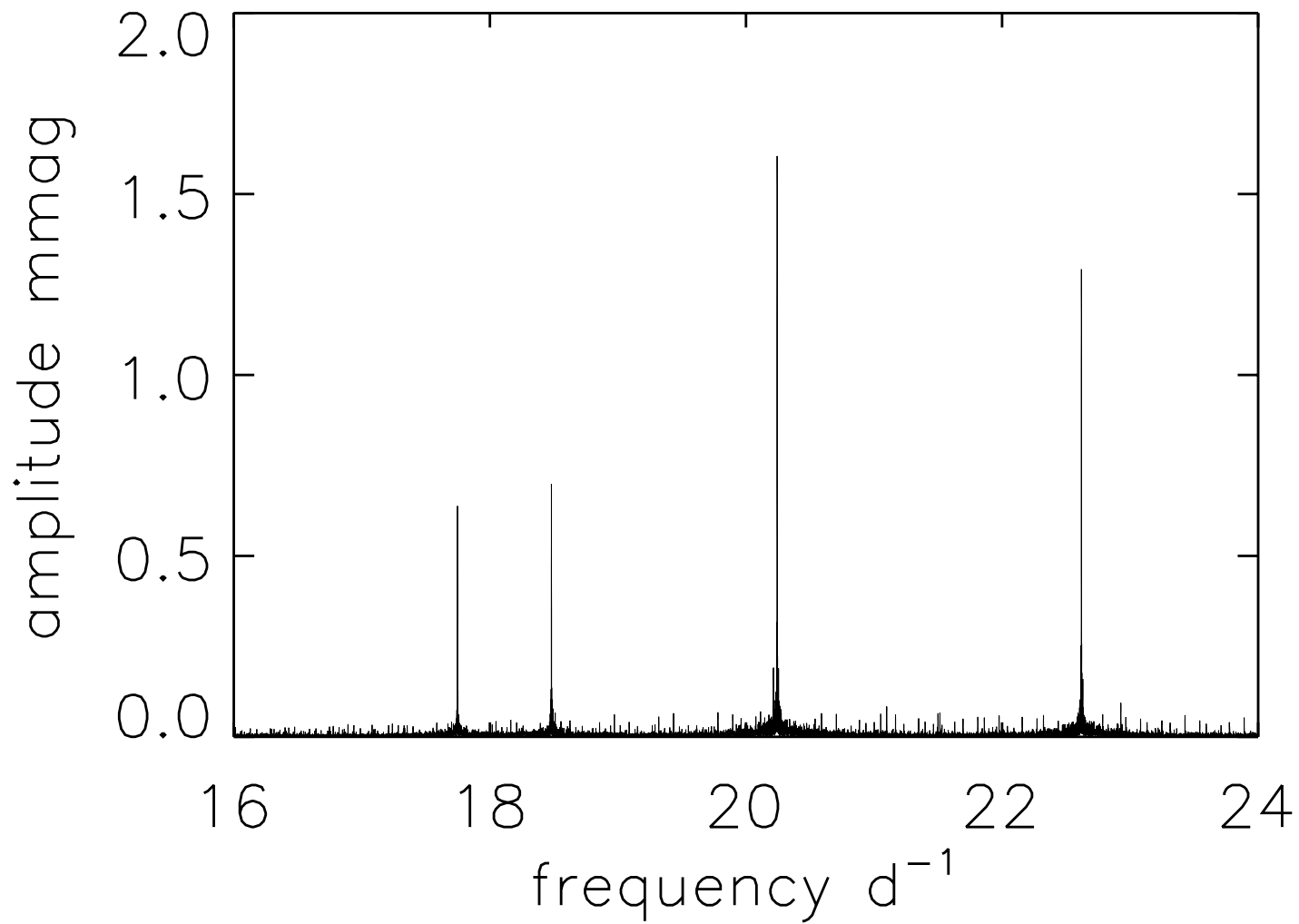
$$= \sum J_n(\alpha) \cos[(\omega_0 + n\Omega)t + \phi]$$



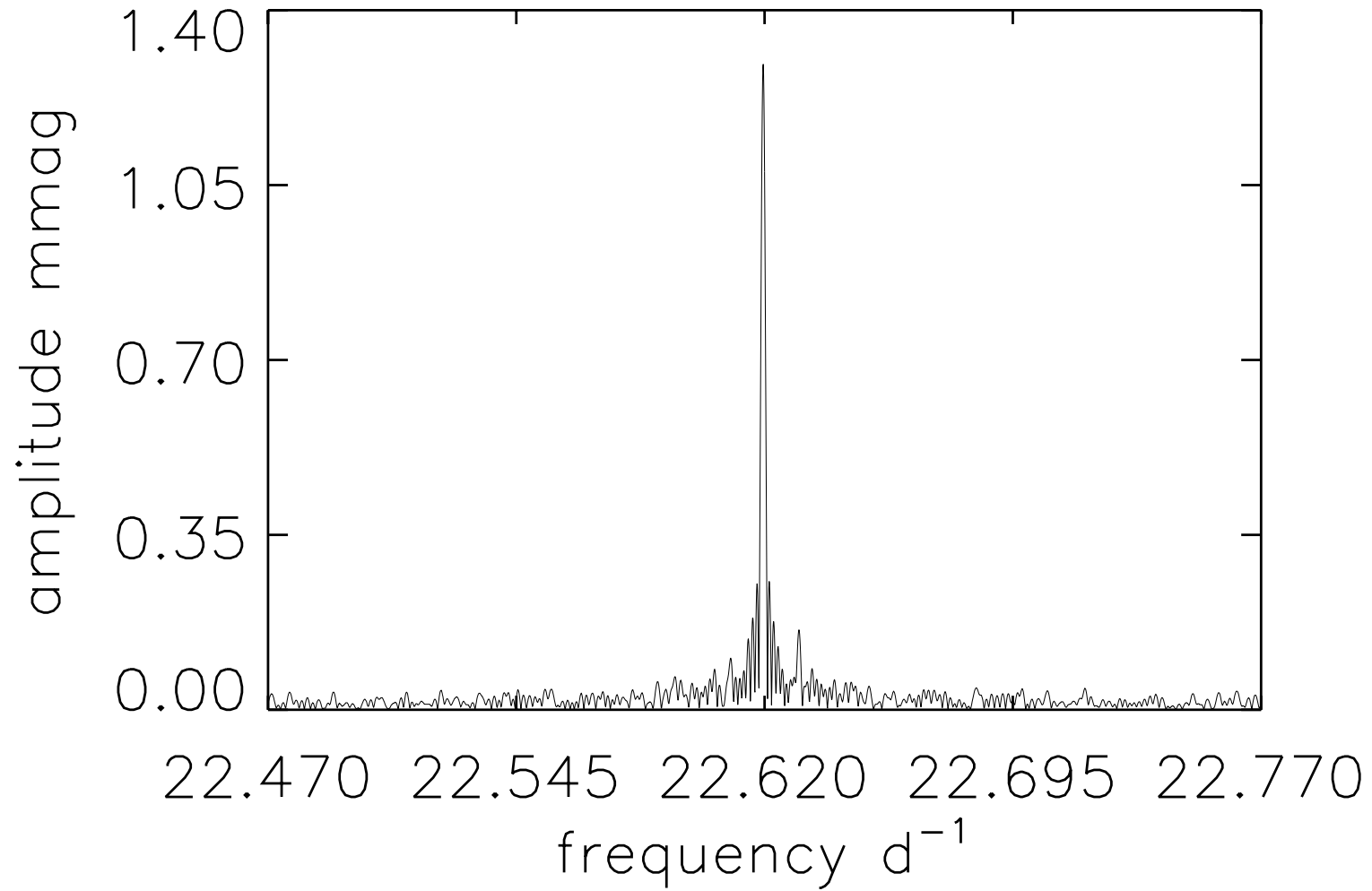
# Bessel functions



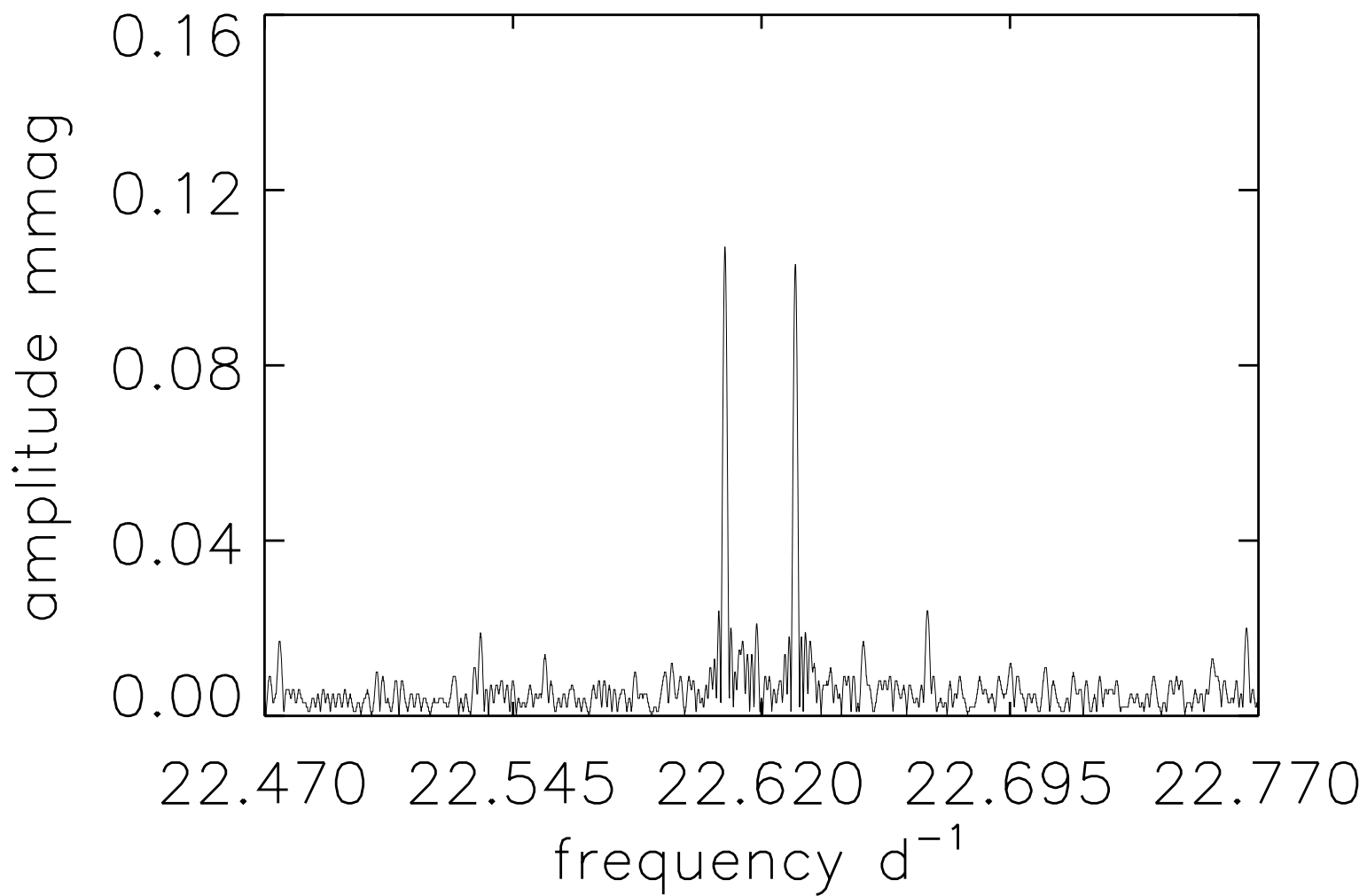
# KIC 4150611



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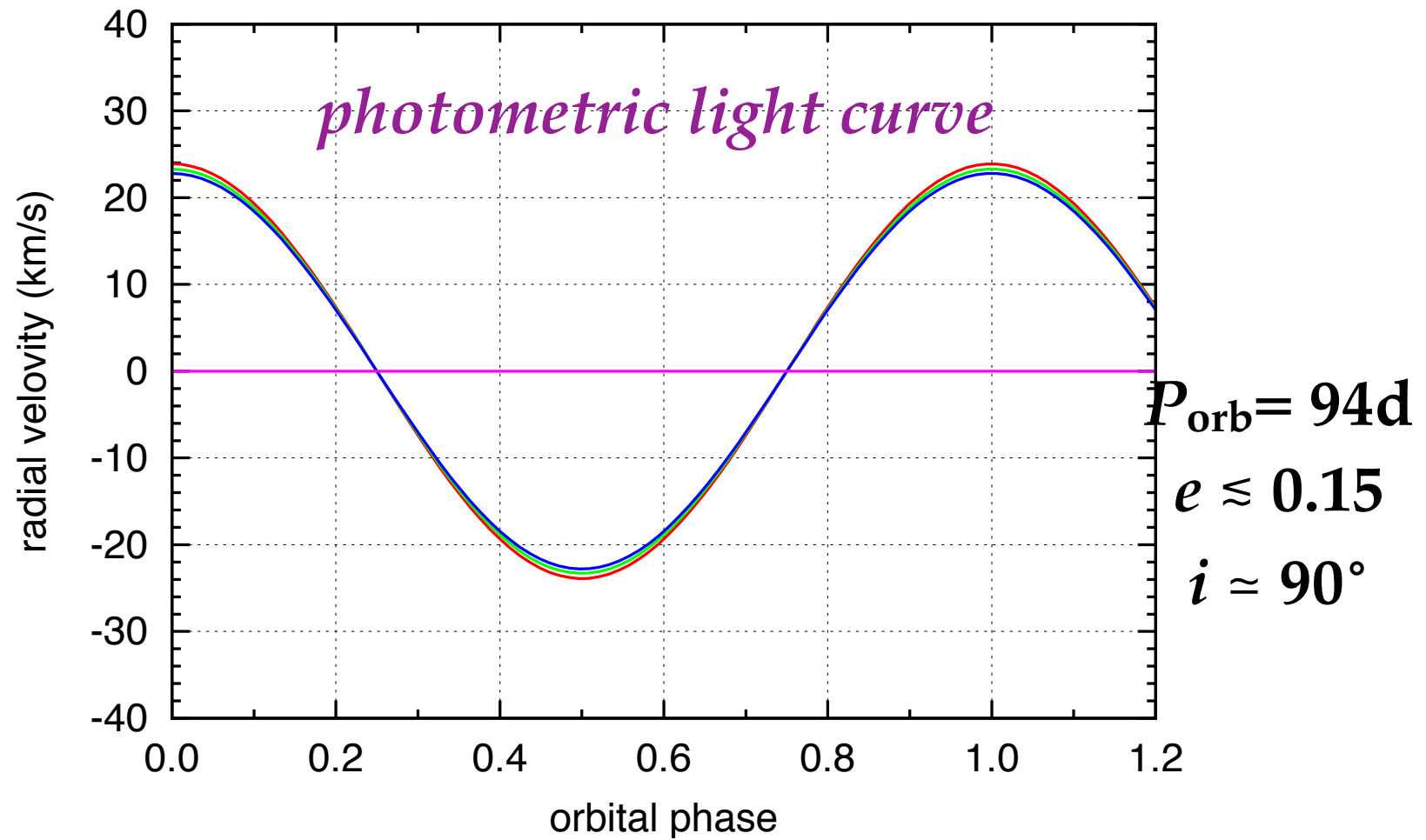
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## binary parameters determined from FM

frequency $\text{d}^{-1}$	$a_1 \sin i$ au	RV amplitude $\text{km s}^{-1}$	$f(m_1, m_2, \sin i)$ $M_{\odot}$
17.7466	$1.30 \pm 0.16$	$23.9 \pm 2.9$	$0.132 \pm 0.116$
18.4805	$1.30 \pm 0.13$	$23.9 \pm 2.4$	$0.133 \pm 0.098$
20.2433	$1.27 \pm 0.05$	$23.3 \pm 1.0$	$0.124 \pm 0.035$
22.6196	$1.24 \pm 0.06$	$22.8 \pm 1.1$	$0.116 \pm 0.034$

# KIC 4150611





# Exoplanet hunting

- photometry

  - ★ FM of pulsation  $\Rightarrow$  the mass of the planet

  - ★ transit  $\Rightarrow$  the size of planet

- The mean density of the planet can be determined from the light curve alone !

- Exoplanet hunting for early type stars

# Running Summary

- Luminosity variations of pulsating stars in binary systems show FM caused by orbital motion.
- FT of FM leads to a multiplet separated by the orbital frequency.
- The binary parameters can be derived from FT of the light curve alone.
- It is possible to detect Jupiter-mass planets orbiting  $\delta$ Sct and other pulsating stars with this technique.

## Present work

$$\cos \left[ \omega_0 \left( t - \frac{1}{c} \int_0^t v_{\text{rad}}(t') dt' \right) \right]$$

*time delay*

$$= \cos \{ \omega_0 t - \Phi(t) \}$$

*Phase Modulation (PM)*

$$\Phi(t) := \frac{\omega_0}{c} \int_0^t v_{\text{rad}}(t') dt'$$

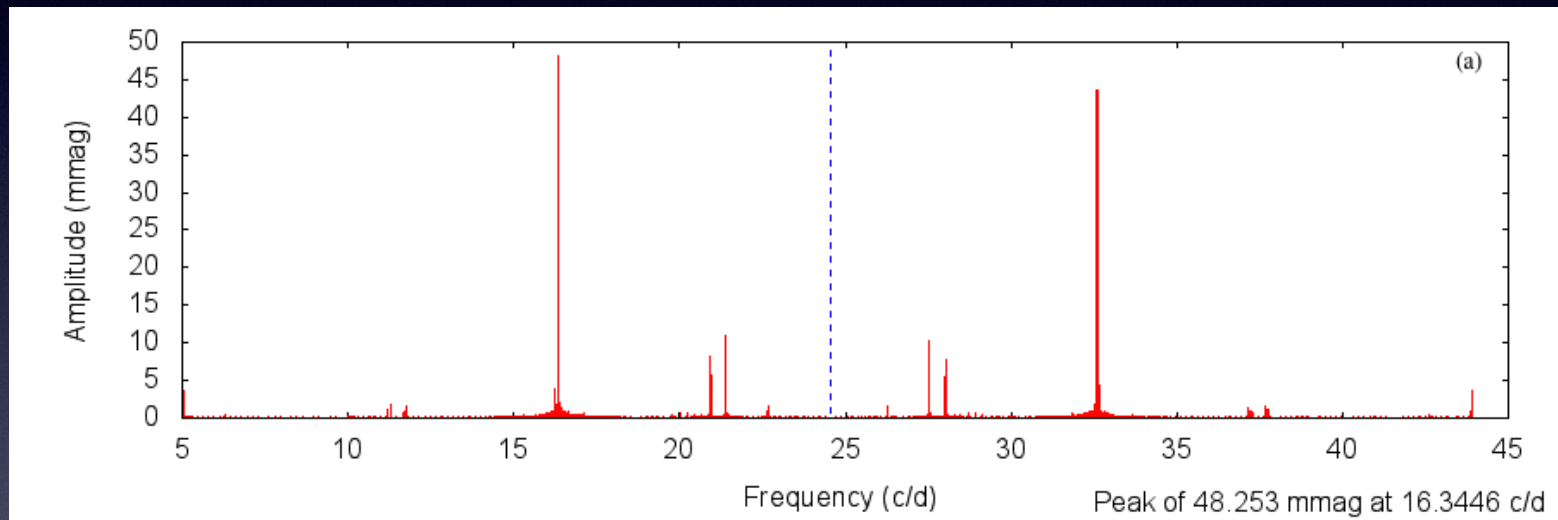
## Time delay

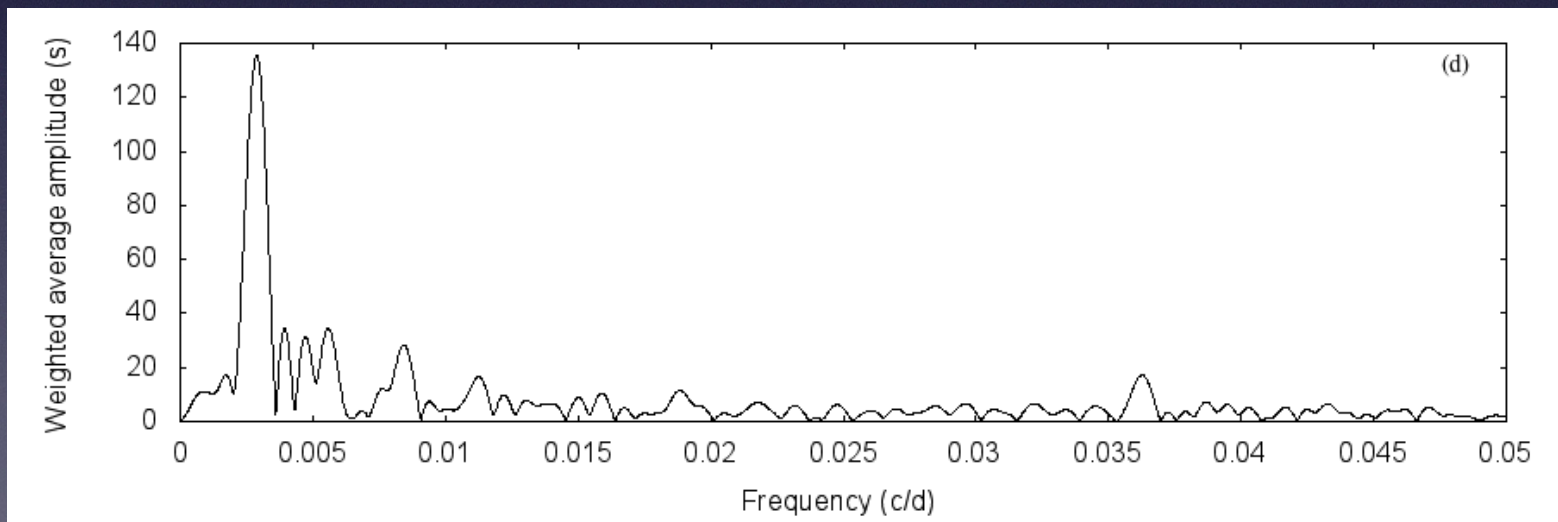
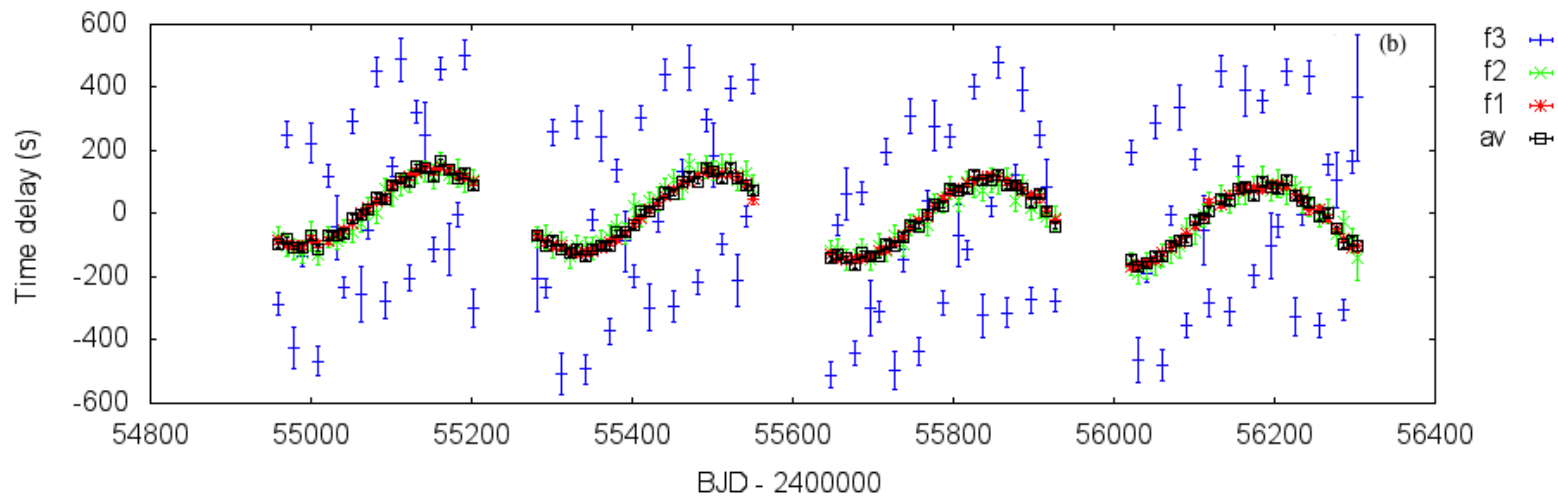
$$\begin{aligned}\tau(t) &= \frac{\Phi(t)}{\omega_0} \\ &= \sum_{n=1}^{\infty} A_n \sin(n\Omega t + \phi_n)\end{aligned}$$

## Radial velocity

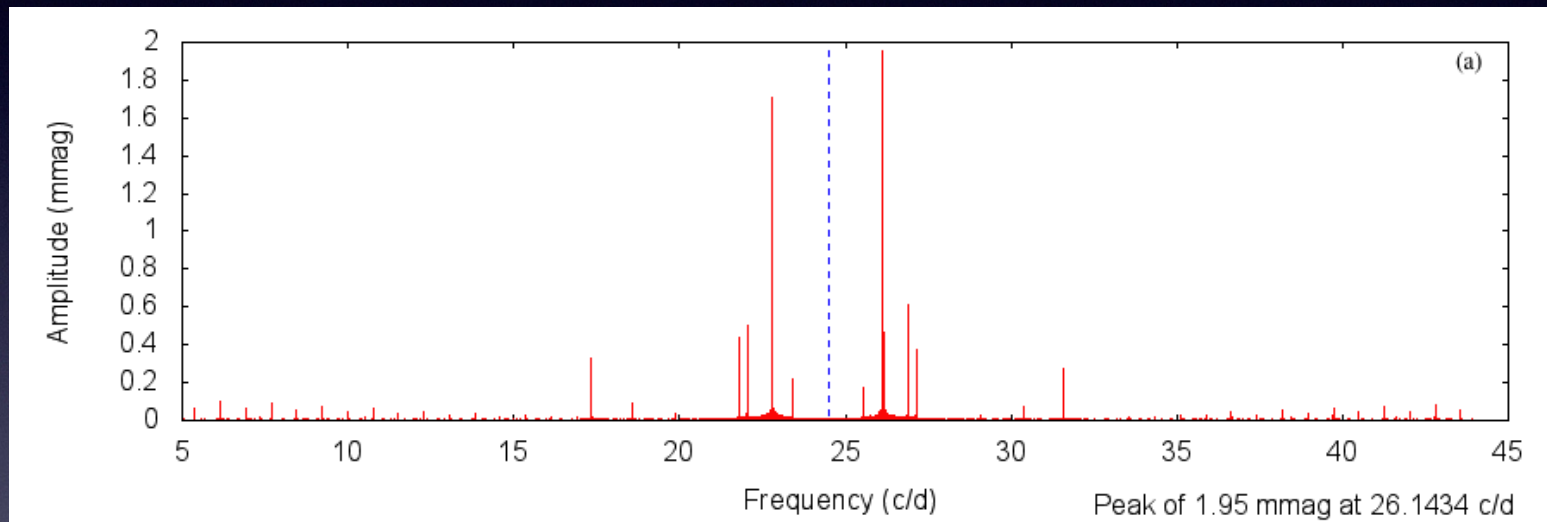
$$v_{\text{rad}}(t) = c \frac{d\tau(t)}{dt}$$

# KIC 11754974

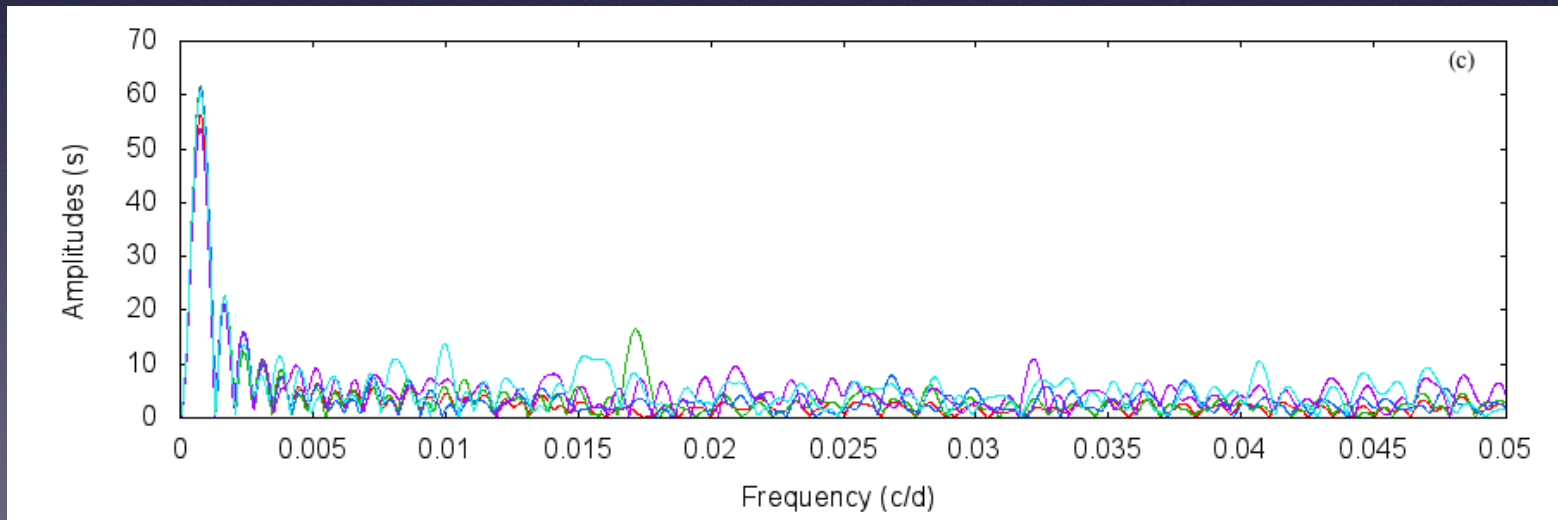
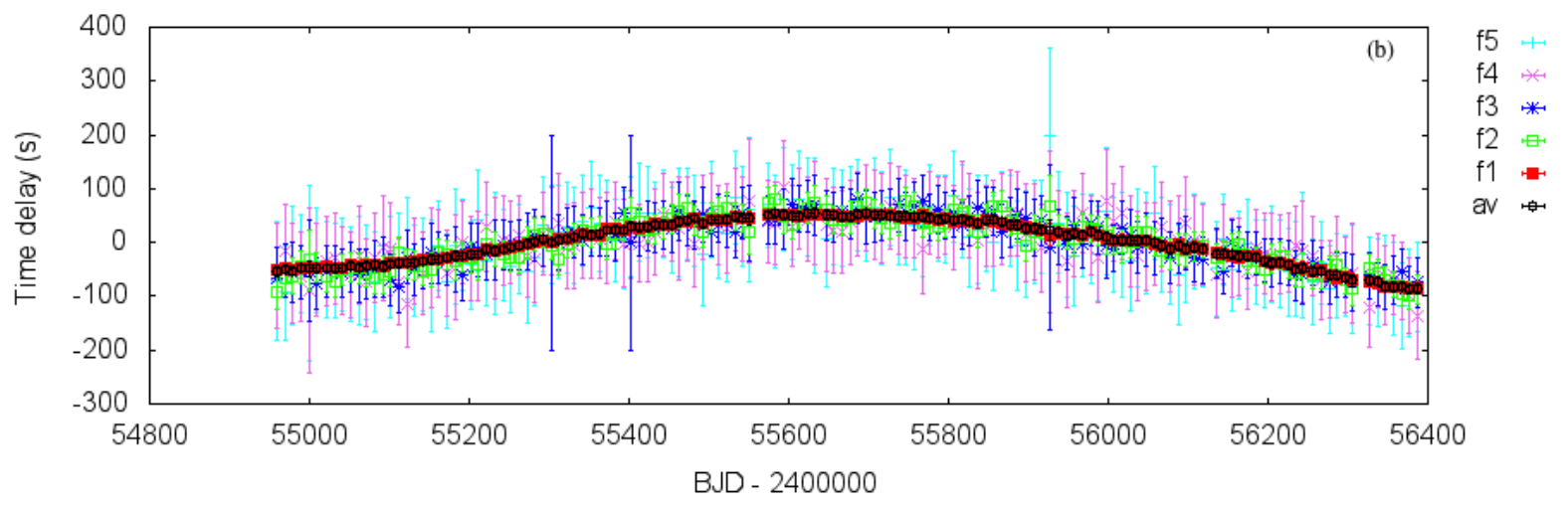




# KIC 7618364

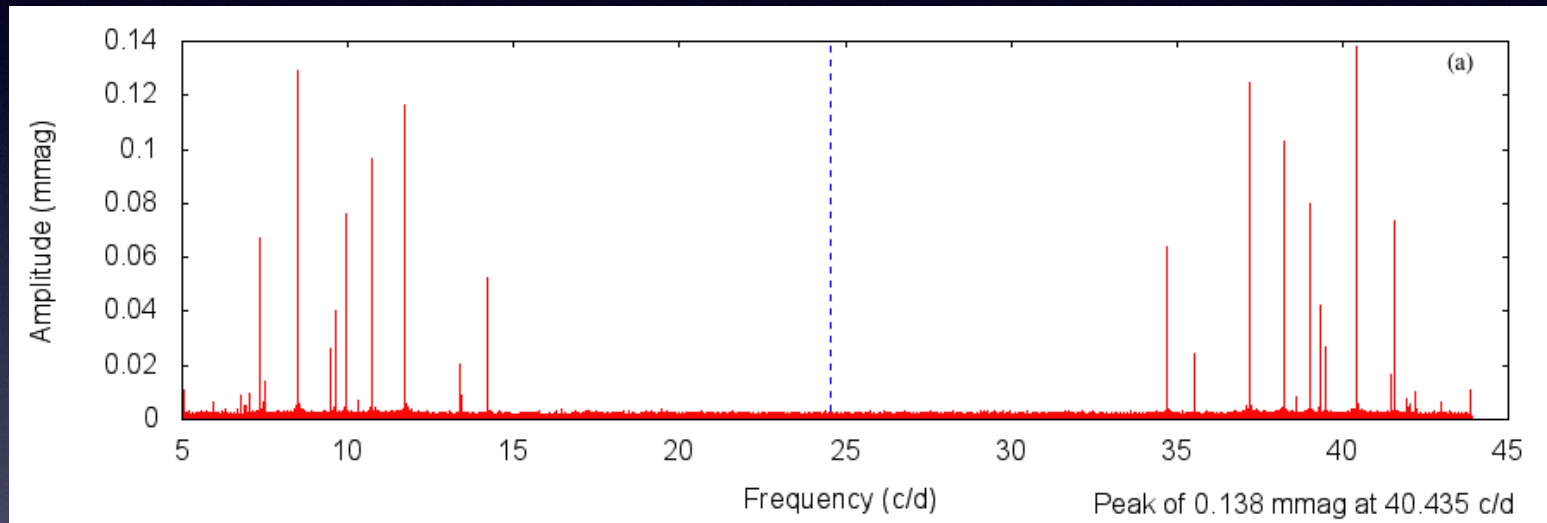


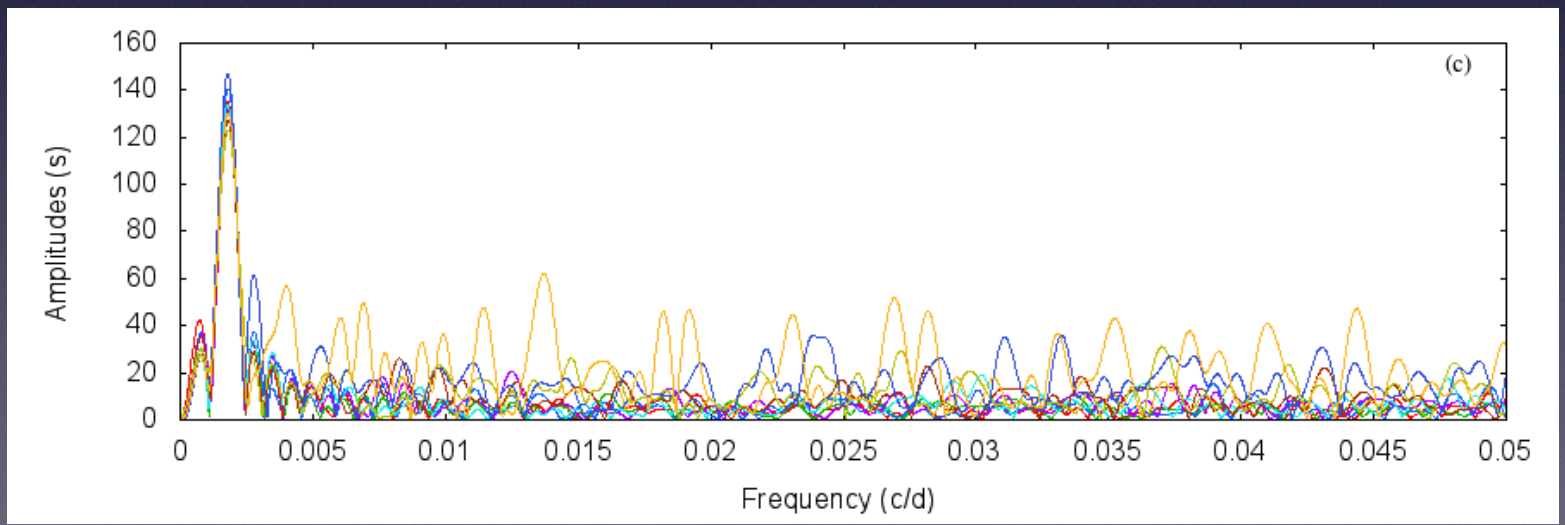
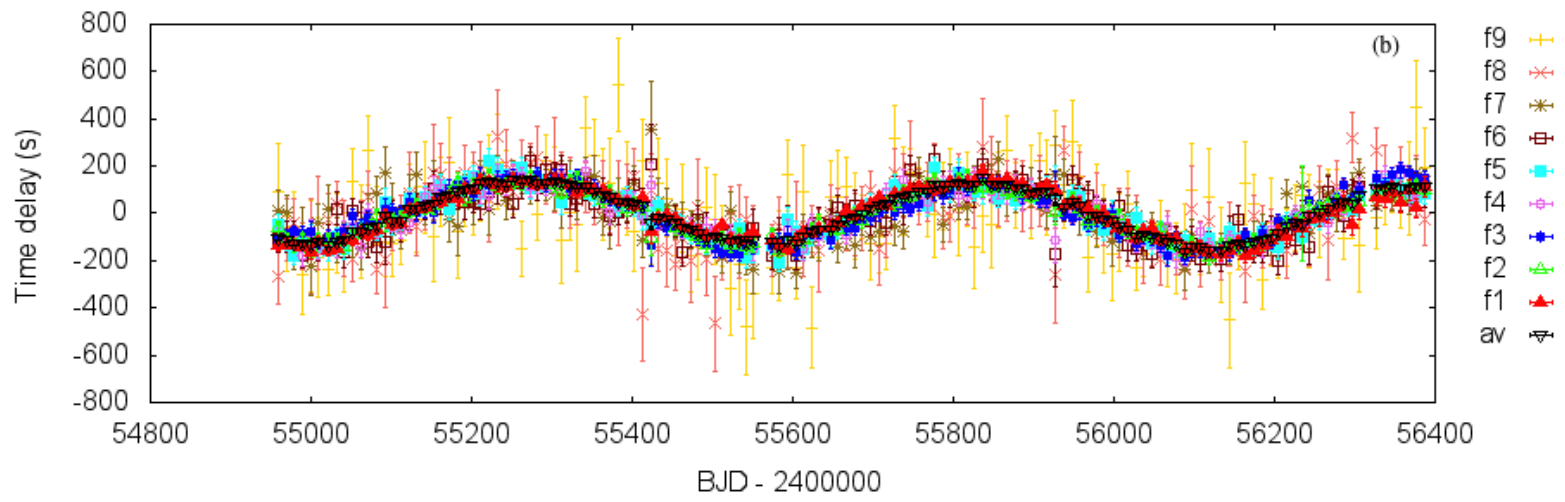
*super Nyquist*



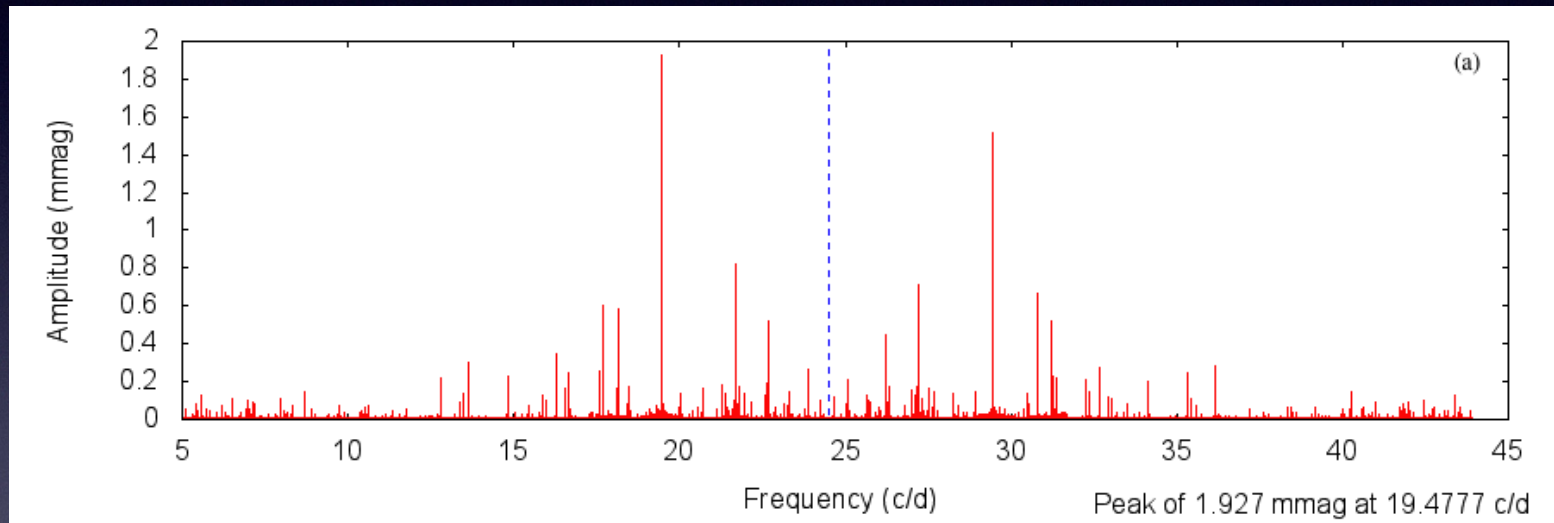


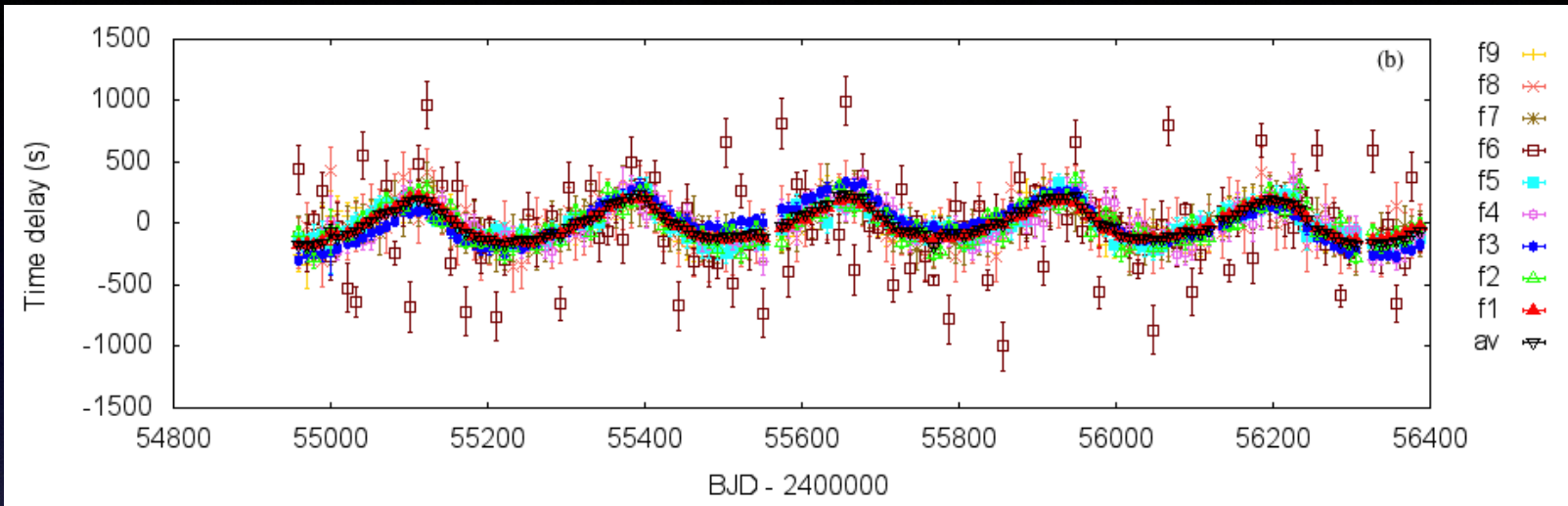
# KIC 11771670



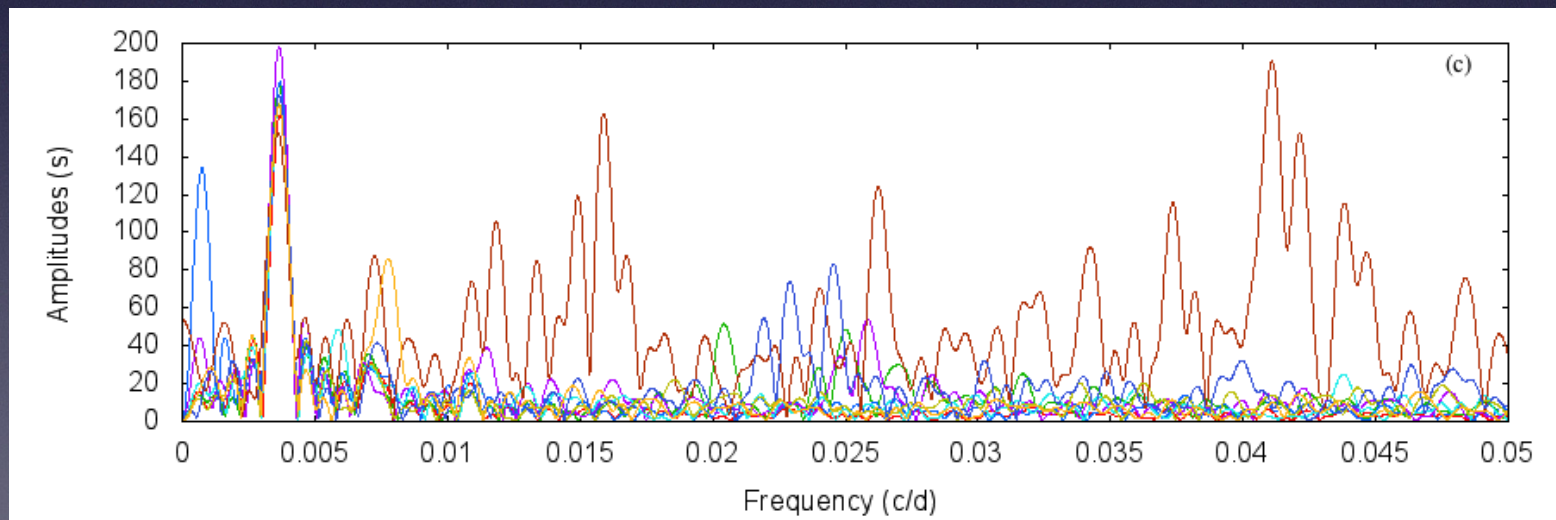


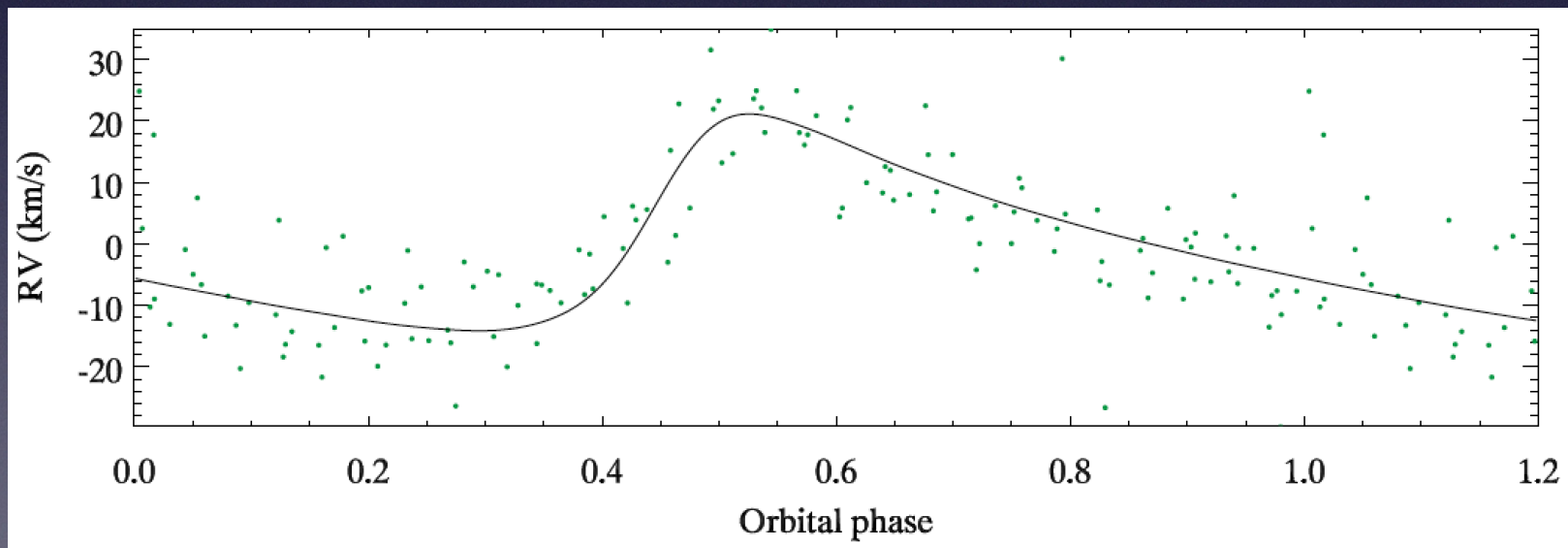
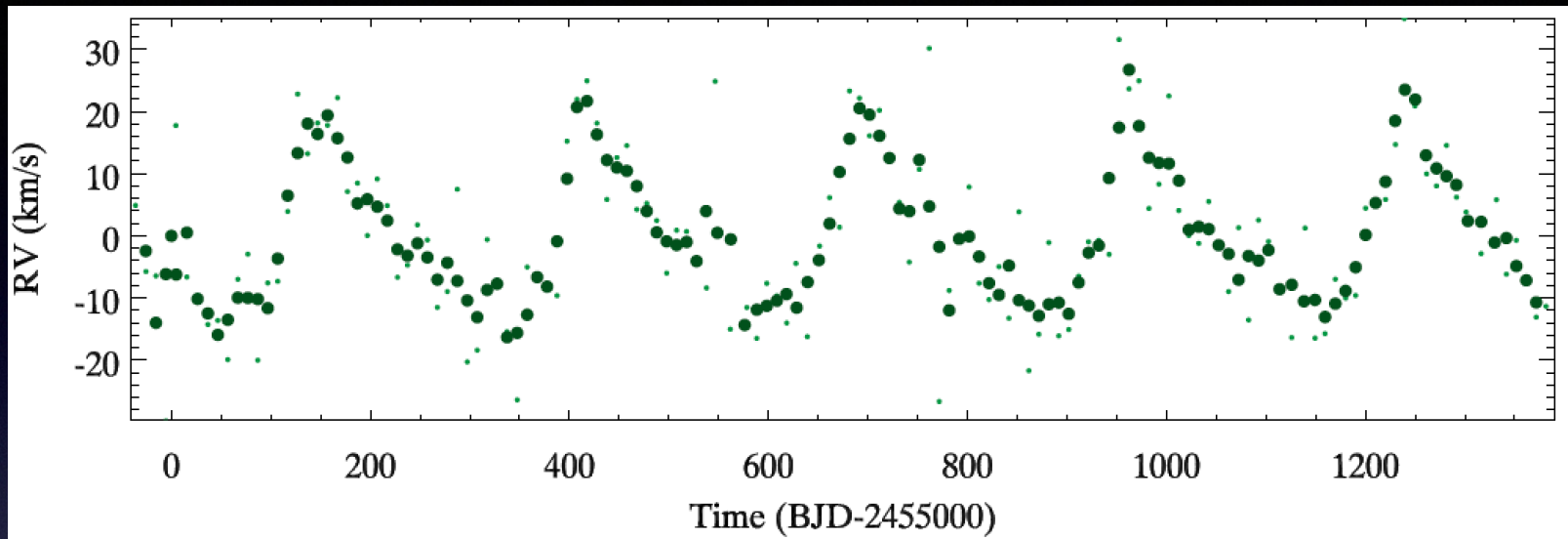
# KIC 9651065





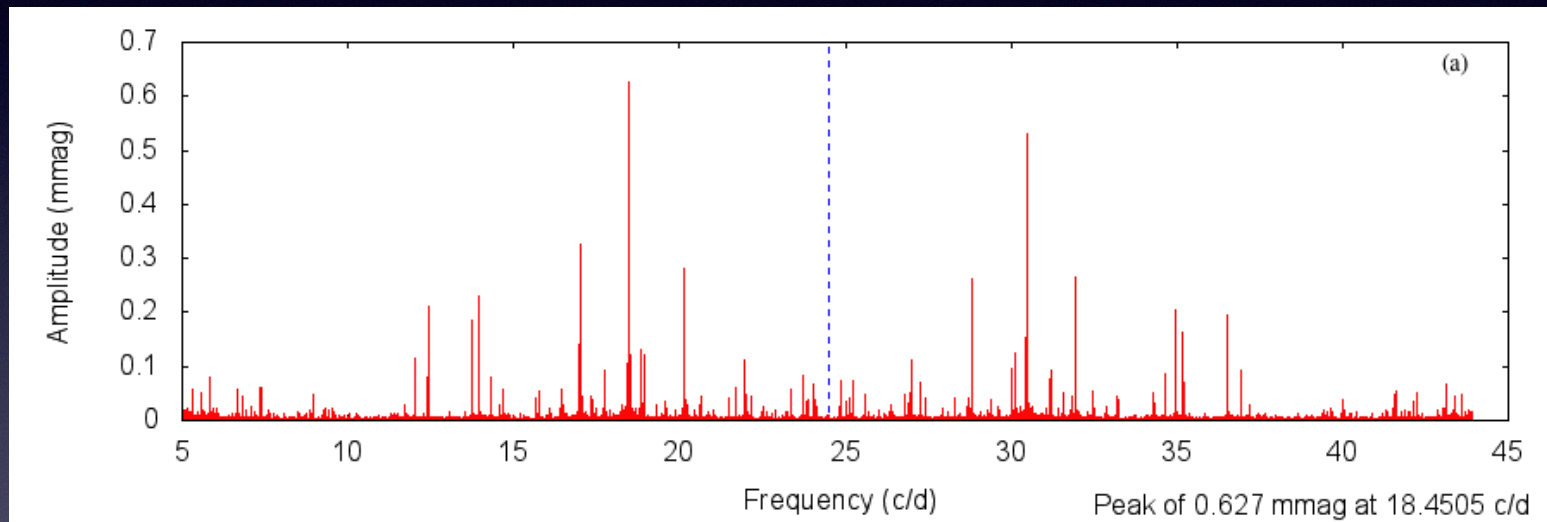
*nonlinear -> highly eccentric orbit*

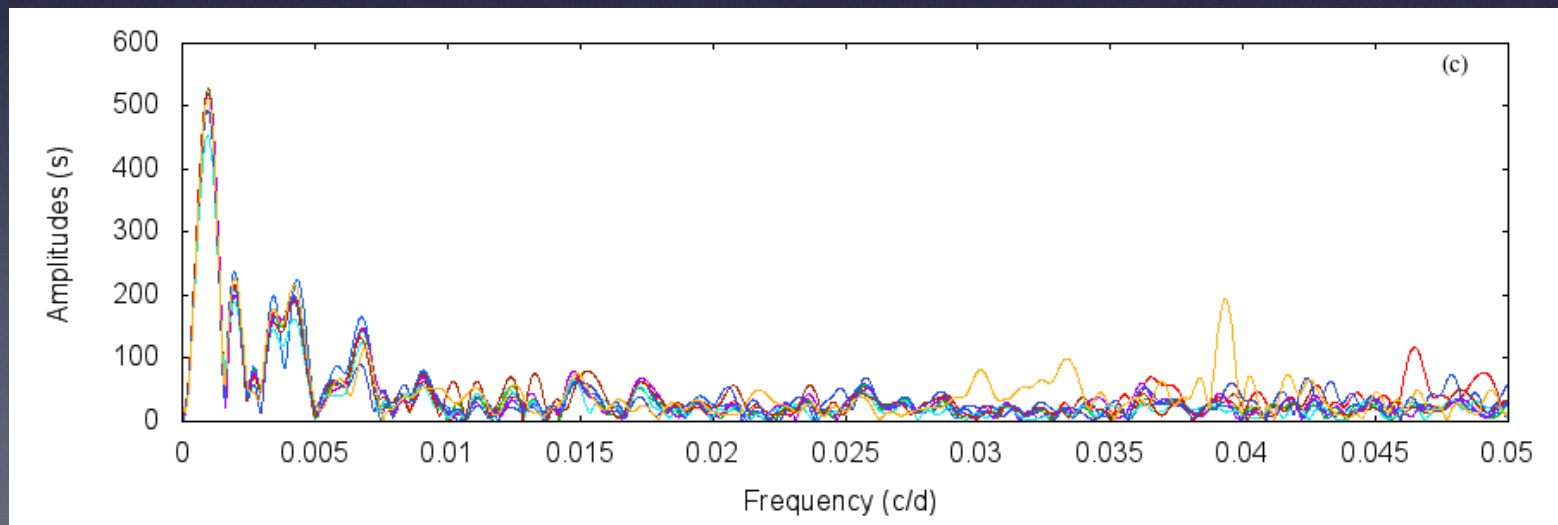
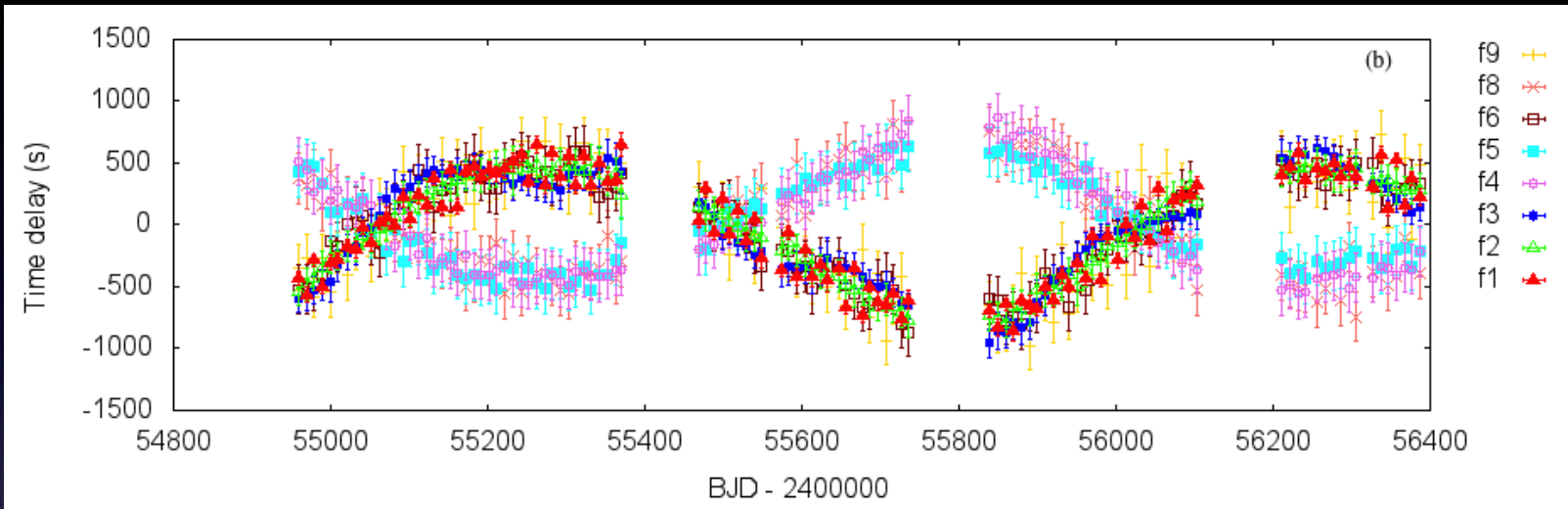




# A binary with two pulsating components

KIC 4471379





## **Advantages of PM**

- Easy for automation**
- Visualisation**
- Time-domain presentation**



# Summary

- Preparation: some basics
- Helioseismology: new eyes to see the invisible solar interior
- Sounds of stars: Asteroseismology and Keplerian revolution
- Interesting new techniques inspired by Kepler
  - Super-Nyquist asteroseismology
  - FM stars