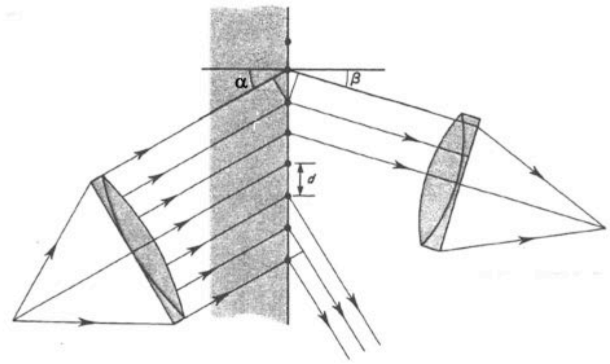
9. Spectrograph Design

9.1 The grating equation

The grating equation gives the condition for constructive interference of the diffracted light from the grating.

The diagram to the right shows collimated light incident on the grating at an angle α and it is diffracted at an angle β. For a diffracted order to exist, the contributions from each wire must be in phase. That is, the waves from each wire must be a multiple of 2π radians to be in phase.



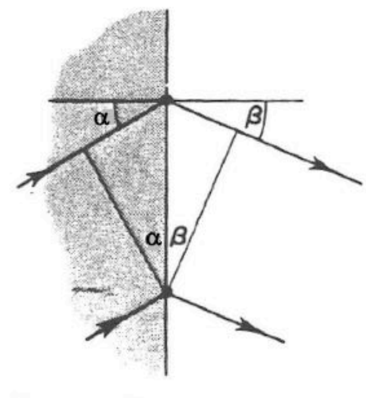
Notes: In this diagram the grating is assumed to be a series of parallel wires into the plane of the slide. This is a simple transmission grating. The grating could also be a series of reflective grooves on a substrate and the equations would be the same.

The path difference for constructive interference requires

$$d\ sin\alpha + d\ sin\beta = m\lambda$$

where m is an integer called the “order number”.

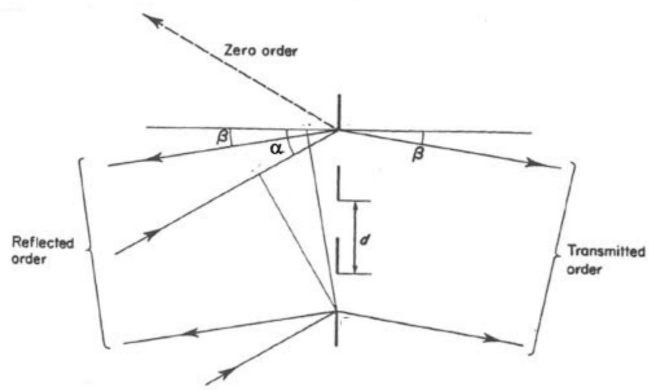
This is the grating equation.



Notes:

The more typical case is that the ruled surface is used as a grating in reflection. The figure to right shows the case of reflective slits that is used as a grating.

The zero order corresponds to a phase difference of zero.

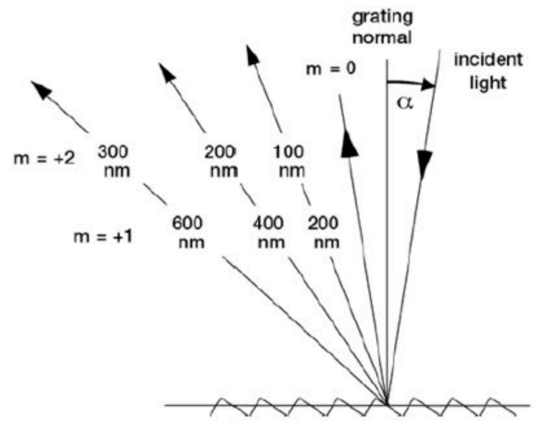


Notes:

If we keep the grating fixed (constant α), then we get a series of angles for β for which the grating equation holds. In first order we would have construction interference for λ, 2λ, 3λ, etc. as shown in the figure.

There are also overlapping wavelengths for second order. Therefore a filter is needed to block out the unwanted orders.

In zero order (m=0) all of the light goes into one direction (no diffraction).



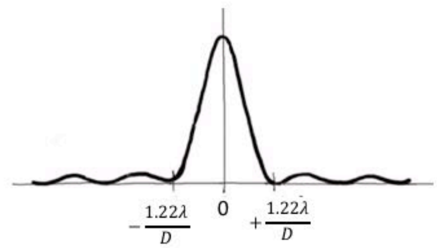
Notes:

9.2 Spectral resolution

By the Rayleigh criterion, two spectral line peaks are considered resolved if the distance between them is larger than the first zero in the Airy disk.

The figure show the Airy function for a circular aperture with a diameter D.

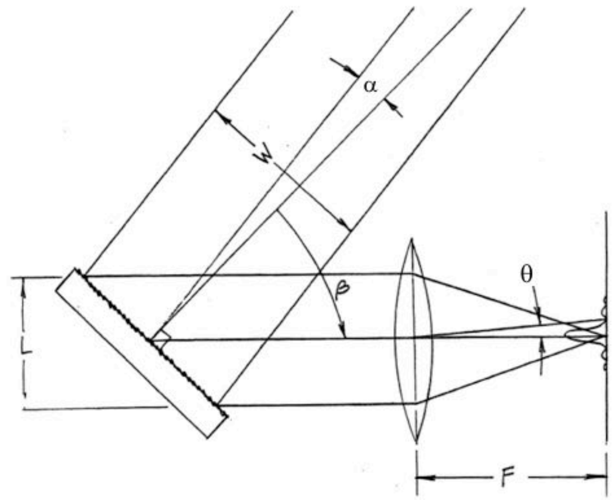
Two spectral lines are consider resolved if the peak of one is at the first zero or further away. This would be at an angle of 1.22λ/D (in radians).



Notes:

Consider a spectrograph with a collimated beam that is diffracted with by grating and an image of the spectral line is formed by a lens of focal length F. The spectral line will have a line profile given by the Airy function.

$$\theta = d\beta = \frac{1.22\lambda}{L} \approx \frac{\lambda}{L} = \frac{\lambda}{W cos\beta}$$



Notes:

The angular width of the spectral line to the first zero of the Airy function is

$$\theta \approx \frac{\lambda}{L} = \frac{\lambda}{W cos\beta}$$

Differentiating the grating equation with respect to β gives

$$cos\beta \; d\beta = \frac{m}{d} d\lambda$$

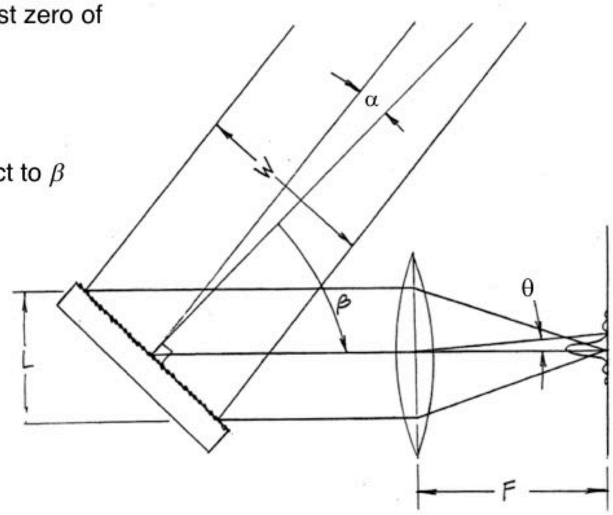
Two wavelengths are resolved if $d\beta = \theta$.
Then

$$\frac{\lambda}{W cos\beta} = \frac{m \, d\lambda}{d \; cos\beta}$$

Then it follows that

$$\frac{\lambda}{d\lambda} = R = \frac{mW}{d} = \mathrm{mN}$$

where N is the total number of grooves in the grating.



Notes:

The equation gives the theoretical resolving power of the grating. In most cases the resolving power of the spectrometer will be limited by the slit width as will be shown below.

9.3 Angular dispersion

The equation for angular dispersion gives the angular separation obtained for two wavelengths separated by $d\lambda$. Differentiating the grating equation, holding α constant, gives

$$\cos\beta \, d\beta = \frac{m}{d} d\lambda$$

from which it follows that

$$\frac{d\beta}{d\lambda} = \frac{m}{d \cos\beta}$$

Substituting for m/d, we get angular dispersion:

$$\frac{d\beta}{d\lambda} = \frac{\sin\alpha + \sin\beta}{\lambda \cos\beta}$$

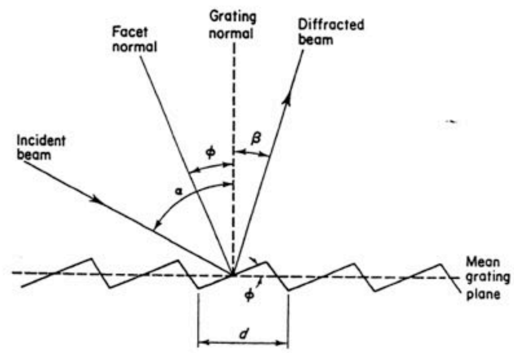
Notes:

9.4 Blazed gratings

By modifying the groove of a reflective grating it is possible to put most of the light into one diffracted order. This is done by "blazing" the grating, that is to cut the grooves in the grating at an angle.

In the figure, the blaze angle is ϕ, and the energy in the diffracted order goes off at an angle of $(\beta + \phi)$ with respect to the normal of the groove surface.

In the case where $\phi = 0^{o}$ (unblazed) much of the energy goes into the 0^{th} order. By choosing the right blaze angle most of the energy can be directed into the direction defined by the angle β. Therefore blazed gratings are always used.

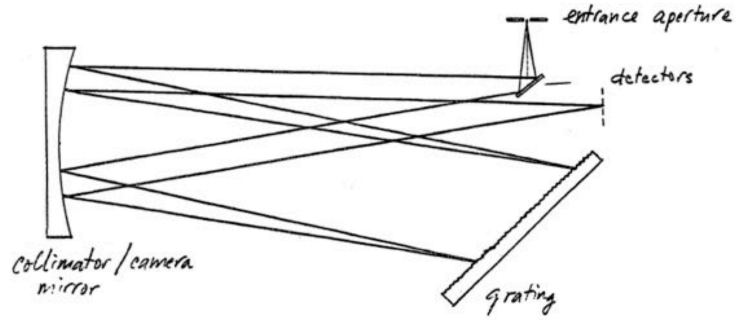


Notes:

This figure shows a simplified spectrograph design called the Littrow. In this design, $\phi = \alpha \approx \beta$ and then from the grating equation we get

$$2\ sin\phi = \frac{m\lambda_B}{d}$$

where λ_B = the blaze wavelength in order m.



This equation specifies the blaze angle corresponding to λ_B. We can see from this equation that a grating blazed for λ_B is also blazed for $\lambda_B/2$, $\lambda_B/3$, … in the 2nd, 3rd, ... orders.

When specifying a grating, the only parameters needed are the number of lines per mm (*d*) and the blaze angle (ϕ).

Notes:

9.5 The spectrograph tradeoff equation.

In a practical spectrograph design, the resolving power ($R = \lambda/\Delta\lambda$) achieved is much less than the theoretical resolving power given by *mN*.

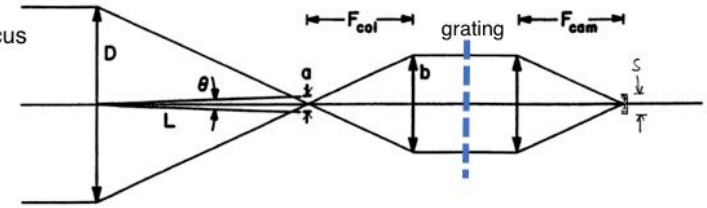
The resolving power is a function of the order number (*m*), wavelength (λ), diameter of the telescope (D_{tel}), entrance aperture in arcsec (*x*), collimator size (D_{col}), grating groove spacing (*d*) and the blaze angle (ϕ). For a given telescope diameter and desired resolving power and slit size, the tradeoff equation tells you how large the collimator size must be. This drives the size of the instrument (and the cost).

To build a practical instrument, the designer must iterate with R, x, and D_{col} to optimize the design in order to achieve the scientific objectives. Another way to think about it is that given the scientific objectives, it is necessary to iterate on R, x, and D_{col} find optimize performance at a minimum of cost.

Notes:

The quantity a/L is the slit length in radians, and x is the slit length in arcsec.

a and s are the slit length at the telescope focus and at the detector and is equal to one resolution element.



We have

$$s = F_{cam} d\beta = F_{cam} \frac{d\beta}{d\lambda} \Delta\lambda \qquad (1)$$

where F_{cam} is the camera focal length, $d\beta/d\lambda$ is the angular dispersion, and $\Delta\lambda$ is one resolution element in wavelength.

$x = \frac{a}{L} \frac{1}{0.485\times10^{-5}}$ (arcsec)

$$\frac{x}{a} = \frac{1}{0.485\times10^{-5} L} = \frac{1}{f_{tel}\, D_{tel}\, 0.485\times10^{-5}} = \frac{206265}{f_{tel} D_{tel}}$$

(= "plate" scale, arcsec/mm)

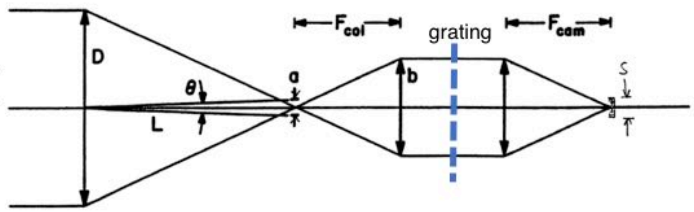
$F_{col} = f_{col}\, D_{col}$, $\quad F_{cam} = f_{cam}\, D_{col}$

$f_{tel}, f_{col}, f_{cam}$ are the telescope, collimator, and camera f/no.

Notes:

The f/no of the telescope, collimator, and camera is the same, so

$$\frac{x}{a} = \frac{206265}{f_{tel}D_{tel}} = \frac{206265}{f_{cam}D_{tel}} = \frac{206265}{D_{tel}}\frac{D_{col}}{F_{cam}}$$



Solving for F_{cam} we get:

$$F_{cam} = \frac{206265}{D_{tel}}\frac{D_{col}\, s}{x} \qquad (2)$$

where we assume the slit is imaged onto the detector ($a = s$).

$x = \frac{a}{L}\,\frac{1}{0.485\times10^{-5}}$ (arcsec)

$$\frac{x}{a} = \frac{1}{0.485\times10^{-5}L} = \frac{1}{f_{tel}\,D_{tel}\,0.485\times10^{-5}} = \frac{206265}{f_{tel}D_{tel}}$$

(= "plate" scale, arcsec/mm)

$F_{col} = f_{col}\,D_{col}$, $\quad F_{cam} = f_{cam}\,D_{col}$

$f_{tel}, f_{col}, f_{cam}$ are the telescope, collimator, and camera f/no.

Notes:

Using equations (1) and (2) we get:

$$R = \frac{206265\ D_{col}}{x} \frac{m\lambda}{D_{tel}\ (d\ cos\beta)}$$

Substitute $m\lambda$ with the grating equation to get

$$R = \frac{206265\ D_{col}}{x} \frac{(sin\alpha + sin\beta)}{D_{tel}\ (cos\beta)}$$

This is the tradeoff equation.

In the case of the Littrow design, we have $\phi = \alpha \approx \beta$ so that

$$R = \frac{206265\ D_{col}}{x} \frac{2\ tan\phi}{D_{tel}}$$

where x is the slit width in arcsec or

$$R = \frac{D_{col}}{x_r} \frac{2\ tan\phi}{D_{tel}}$$

where x_r is the slit width in radians.

One has to tradeoff slit width against the collimator diameter in practical instruments.

Notes:

9.5 Example of the SpeX instrument on the IRTF.

SpeX is a low and moderate resolution spectrograph. As a facility instrument it was designed

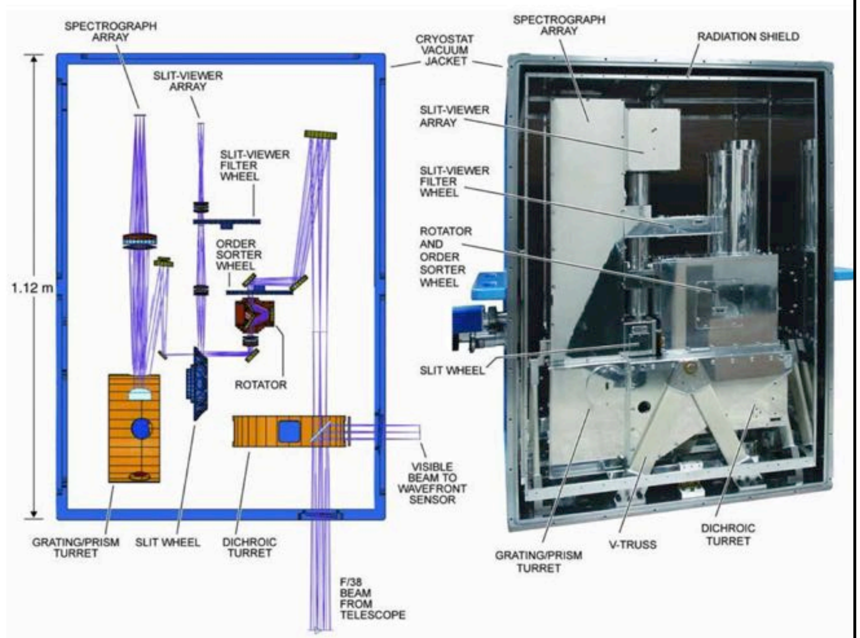
- to provide spectra of solar system objects, star forming regions, the interstellar medium, and star in all stages of evolution.
- maximum spectral coverage at 0.8-2.5 μm and 1.9-5.5 μm for R=2500 with 0.3"x15" slits [for studies of star forming regions and stars, interstellar medium]. Uses cross-dispersion to separate orders.
- low resolution spectra at 0.8-2.5 μm (R=150) [for studies of asteroids, brown dwarfs] with 0.3"x15" or 0.3"x60" slits. Uses a prism.
- has a slit viewer to allow guiding in the IR.

Notes:

Rayner, J. T. et al. (2003). "SpeX: A Medium-Resolution 0.8-5.5 Micron Spectrograph and Imager for the NASA Infrared Telescope Facility." Publications of the Astronomical Society of the Pacific **115**: 362-382.

Layout of the instrument.

Slit width of 0.3 arc for highest resolution. Collimated beam diameter is 42 mm. Gratings have a blaze angle of 10.2° or 12.7°.



Notes

Rayner, J. T. et al. (2003). "SpeX: A Medium-Resolution 0.8-5.5 Micron Spectrograph and Imager for the NASA Infrared Telescope Facility." Publications of the Astronomical Society of the Pacific **115**: 362-382.

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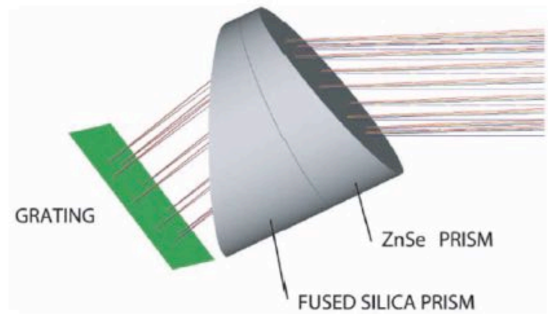
The overlapping orders of the grating are separated using a prism. The collimated light passes through the grating twice. This method achieves a compact optical design.



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Example of cross-dispersed spectrum.

This is a subtraction of two exposures where the object has been moved along the slit in between the exposures. So one stellar continuum is positive (white) and the other is negative (black).

Cross-dispersion by the prism separates the orders. The orders observed are limited by a bandpass filter, other more orders would appear. The optical design uses as much of the IR array as possible.

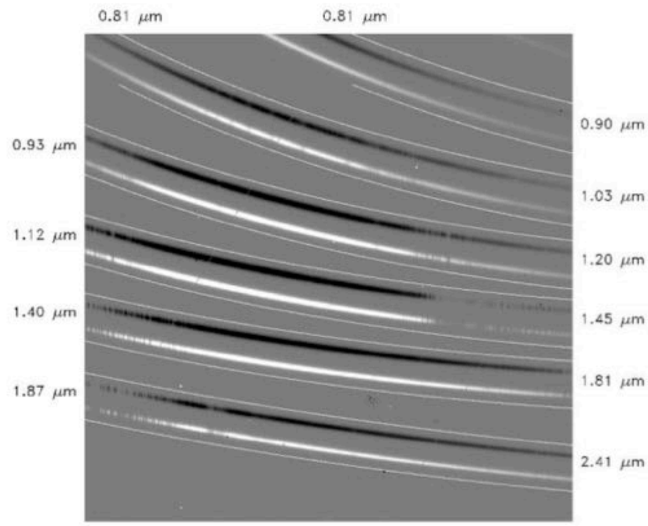

FIG. 1.—SXD mode. Short-wavelength (0.8–2.4 μm) cross-dispersed format. Shown is an object minus sky image where a star has been nodded 7″.5 in the 15″.0 slit. The white lines indicate the top and bottom of the slit. Note the low-transmission telluric features.

Notes:

Rayner, J. T. et al. (2003). "SpeX: A Medium-Resolution 0.8-5.5 Micron Spectrograph and Imager for the NASA Infrared Telescope Facility." Publications of the Astronomical Society of the Pacific **115**: 362-382.