16 Feb. 2018

Introduction to IR Astronomy, Alan Tokunaga

Problem set. Submit the answers by email to tokunagaa001@gmail.com by 5pm on Weds. Feb. 21.

1. Derive the conversion formula:

$$F_\lambda = 3\times10^{14}\ F_\nu/\lambda_{\mu m}^2$$

where $[\,F_\lambda] = Wm^{-2}\mu m^{-1},\ [\,F_\nu] = Wm^{-2}Hz^{-1}, [\nu] = Hz,\ [\lambda_{\mu m}] = \mu m$

2. Assume you want to observe a G2 star (Teff = 5800K) behind a dark cloud with extinction A_v = 7mag. Which would you choose: A 10-m telescope observing at V (0.55 mag) or a 1-m telescope observing at K (2.2 μm)? Justify your answer.

3. Compute the limiting flux density in Jy for the Thirty-Meter Telescope and the James Webb Space Telescope for a resolving power (λ/Δλ) of 5.0, $2.5\text{x}10^3$, and $1\text{x}10^5$ at a wavelength of 2.2 μm and 11.5 μm. Assume a 0.45"x0.45" field of view and a pixel size of 0.15" for both telescopes. For the calculation assume the following:
   a. Near-IR night sky brightness given in slide 3-10. Assume the darkest sky case for Maunakea.
   b. Mid-IR sky brightness given in slide 4-12.
   c. The background in space given in slide 4-18.
   d. A collecting area of 700 m^2 for the TMT and a collecting area of 25 m^2 for JWST.
   e. A time on source of 30 minutes.
   f. Detector read noise of 7 electrons at 2.2 μm and 14 electrons at 11.5 μm.
   g. Detector dark current of 0.05 electrons/sec at 2.2 μm and 0.2 electrons/sec at 11.5 μm.
   h. Detector full well of 200,000 electrons.
   i. Instrument throughput of 0.20 (this includes the detector quantum efficiency).
   j. Signal-to-noise = 5.

Note that the diffraction-limit of the JWST at 11.5 μm is 0.45" and the seeing at Maunakea is close to 0.45" most of the time. For simplicity, we assume a constant image size in this problem.

What do you conclude from these calculations regarding the advantages and disadvantages of observing from the ground?

11 March 2018

Problem 1.

The flux from a source can be expressed as:

$$F = F_\lambda \Delta\lambda = F_\nu \Delta\nu \qquad W m^{-2}$$

from which we get:

$$F_\lambda = F_\nu \frac{\Delta\nu}{\Delta\lambda}$$

Since $\nu = c/\lambda$ we have $d\nu = \frac{c}{\lambda^2} d\lambda$ (the negative sign is dropped since we are interested only in the absolute value of $d\nu$). Then

$$F_\lambda = \frac{c}{\lambda^2} F_\nu$$

Expressing *c* and λ in units of micrometers, we get

$$F_\lambda = \frac{3\times 10^{14}}{\lambda_{\mu m}} F_\nu \qquad W\ m^{-2}\ \mu m^{-1}$$

2. The flux density we observe from the object is proportional to $e^{-\tau}B_\lambda(5800\ K)A_{tel}$, where τ is the optical depth, B_λ is the Planck function, and A_{tel} is the telescope area.

$$A_\lambda = -2.5\log(e^{-\tau_\lambda}) = 1.086\ \tau_\lambda$$

From the table on slide 11-3, $A_K = 0.114 * A_V$.

We therefore get:

λ	A_{tel}	τ_λ	$B_\lambda(5800\ K)$	$e^{-\tau}B_\lambda(5800\ K)A_{tel}$
V (0.55 μm)	78.6 m^2	7/1.086	2.63×10^7	3.28×10^6
K(2.2 μm)	0.786 m^2	0.798/1.086	1.11×10^6	4.18×10^5

B_λ is the Planck function with units W m^{-2} μm^{-2} sr^{-1}

So this shows we get about 8 times more signal at the V band and so it is better to observe at V.

In a realistic case, such as writing an observing proposal, we would integrate over the filter bandpass and include the sky background and detector read noise and dark current. But the result will be the same since the difference is large and the sky background is higher at K.

3. This problem is harder than I intended it to be but some of you did very well on it. I gave full credit for this problem if you were able to set up the calculation properly, used the right numbers for the background, and got answers that were not too many orders of magnitude off.

Use the signal to noise equation on slide 6-3 and assume $n_b >> n_p$. Compute the background in electrons/sec/pixel using the equation on slide 6-5:

$$B = A\Omega \int \lambda\varepsilon(\lambda)B_\lambda(\lambda, T)Q(\lambda)d\lambda/hc,$$

The quantity

$$\varepsilon_s(\lambda)B_\lambda(\lambda, T)$$

can be obtained from the slides 3-10, 4-12, and 4-18. To convert mag/square arcsec to a flux density use the table on slide 7-22, which gives the flux density of Vega and Vega is defined to be 0.0 mag. The magnitudes given in slide 3-10 is relative to Vega, and *these are not AB magnitudes*. I have corrected slide 3-10 to make this clear. When the AB mag is given in publications, they will denote it as "m_{AB}". When magnitudes are given relative to Vega, they will usually denote it as "Vega magnitudes".

I obtain the following table for the background per pixel:

	This is n_b (photo-electrons/sec/pixel).			
	TMT	**JWST**		
With R=5	**ground**	**space**		
2.2 mic	7.71E+03	1.02E+00	elec/sec	grey = background limited
11.5 mic	1.45E+09	1.08E+02	elec/sec	yellow = readnoise limited
	TMT	**JWST**		
With R=2500	**ground**	**space**		
2.2 mic	1.54E+01	2.04E-03	elec/sec	
11.5 mic	2.90E+06	2.16E-01	elec/sec	
	TMT	**JWST**		
With R=100 000	**ground**	**space**		
2.2 mic	3.86E-01	5.10E-05	elec/sec	
11.5 mic	7.25E+04	5.39E-03	elec/sec	

I assume an exposure time of 300 sec unless the array becomes saturated (the background electrons will equal 200,000. This is the meaning of the term "full well" of the array. This will happen in all of the cases for the TMT at 11.5 μm. For 2.2 μm, the detector gets saturated in the TMT case for R=5 and is not saturated for R=2500 and 10^5.

I assume 25 sec exposure for 2.2 μm and R=5, 0.07 sec for 11.5 μm and R=2500, and 3.0 sec for 11.5 μm and R=10^5. I don't calculate the case for 11.5 μm and R= 5 since the array needs to be read out faster than 1msec and this is not possible (but you were not expected to know this).

The JWST never gets saturated for 300 sec exposure time.

The table shows the cell in grey color if the pixel background is greater than the read noise and dark current and light orange color if the pixel is read noise limited. Then the source signal in photo-electrons per second can be calculated as shown below using the signal-to-noise equation on slide 6-3.

In the background limited case,

$$SNR = N_T / \sqrt{N_T + n_p(N_b)}$$

Then solve for S using $N_T = S\,t$ and $N_b = n_p t$, where S is the signal in electrons/sec and t is the time for a single exposure. A quadratic equation is obtained:

$$S^2 t - S(SNR)^2 - (SNR)^2 n_p b_p = 0$$

where n_p is the number of pixels and b_p is the background in photo-electrons/sec/pixel.

In the read noise limited case,

$$SNR = N_T / \sqrt{N_T + n_p(N_{rn}^2)}$$

and another quadratic equation is obtained:

$$S^2 t^2 - S(SNR)^2 t - (SNR)^2 n_p N_{rn}^2 = 0$$

Use the quadratic equations to solve for S.

An equation for f_ν can be derived from the signal equation on slide 6-4:

$$S = A \int (f_\lambda / h\nu)\, Q(\lambda) \tau_a(\lambda) d\lambda$$

Solve for S analytically by approximating f_λ as a constant in a small wavelength interval $\Delta\lambda$. Use $\Delta\lambda_{\mu m} = \lambda_{\mu m}/R$ and $\nu = 3\text{x}10^{14}/\lambda_{\mu m}$ to get:

$$S = \frac{A\, \lambda_{\mu m}^2\, f_\lambda\, Q\, \tau_a}{3\times10^{14} h\, R}$$

Substitute

$$f_\lambda = f_\upsilon \frac{3\times10^{14}}{\lambda_{\mu m}^2}$$

and solve for f_ν to get:

$$f_\nu = \frac{SRh}{AQ\tau_a} = 6.627\times10^{-34} \frac{SR}{AQ\tau_a} = 3.31\times10^{-7} \frac{SR}{A} \qquad Jy$$

where $Q\tau_a$ is assumed to be 0.2 (typical throughput).

Using S computed from the quadratic equation, get f_ν as a function of R and A.

This table shows f_ν as a function of the resolving power and telescope area for an exposure time of 300 sec unless the array is saturated. If the array is saturated then use the shorter integration time mentioned previously. Green cells show which telescope has a better sensitivity.

calculate f_nu	This is f_nu (Jy) for S/N=5 for a source in a 0.45"x0.45" area of the sky.				
	700	25		Area of telescope	
With R=5					
	TMT	**JWST**			
	ground	**space**			
2.2 mic	6.24E-07	3.18E-08	Jy	**Comparison is not accurate since the**	
11.5 mic	---	5.98E-07	Jy	**TMT will work at the diffraction limit and**	
				this will lower the background significantly.	
With R=2500					
	TMT	**JWST**			
	ground	**space**		Green = more sensitive	
2.2 mic	4.07E-06	1.30E-05	Jy		
11.5 mic	1.14E-01	2.46E-05	Jy		
With R=100 000					
	TMT	**JWST**			
	ground	**space**			
2.2 mic	2.75E-05	5.22E-04	Jy		
11.5 mic	1.11E-01	9.84E-04	Jy		

Now for the last step. The problem asked for the limiting flux density in 30 minutes of integrating time. Add up n exposures to build up an image and the noise will decrease by $\sqrt{n}$. So compute the limiting flux density by dividing the results in the table by the square root of the number of exposures to build up an image with 30 minutes of integrating time. This is shown in the next table:

Calculate f_nu for 30 minutes integrating time				
With R=5				
	TMT	**JWST**		
	ground	**space**		
2.2 mic	7.36E-08	1.30E-08	Jy	Green = more sensitive
11.5 mic	---	2.44E-07	Jy	
With R=2500				
	TMT	**JWST**		
	ground	**space**		
2.2 mic	1.66E-06	5.32E-06	Jy	
11.5 mic	7.13E-04	1.00E-05	Jy	
With R=100 000				
	TMT	**JWST**		
	ground	**space**		
2.2 mic	1.12E-05	2.13E-04	Jy	
11.5 mic	4.52E-03	4.02E-04	Jy	

The JWST is always better for 11.5 μm. The JWST is also better at 2.2 μm for R=5. The TMT is much better at 2.2 μm for R=2500 and 10^5.

This shows that for the assumptions in this problem, the TMT is better for spectroscopy where the atmosphere is transparent. Unique moderate and high resolution spectroscopy can be done with TMT that no space observatory can match.

In reality, the TMT is planning to use adaptive optics for near-IR and mid-IR observations. Thus the background will be much lower than assumed in this problem. If we consider the diffraction limited spot size for TMT and JWST we have:

	2.2 μm	11.5 μm
JWST	0.085"	0.45"
TMT	0.018"	0.096"

Compared to the 0.45"x0.45" area on the sky assumed in this problem the background for the TMT will be 625 times lower at 2.2 μm and 22 times lower at 11.5 μm. The TMT will be competitive with JWST at 2.2 μm and more sensitive for spectroscopy than calculated in this problem.